Renormalization of the effective Lagrangian with spontaneous symmetry breaking: The $SU(2)$ case

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We study the renormalization of the nonlinear effective $SU(2)$ Lagrangian up to $O(p^4)$ with spontaneous symmetry breaking. The Stueckelberg transformation, the background field gauge, the Schwinger proper time and heat kernel method, and the covariant short distance expansion technology guarantee gauge covariance and incorporate the Ward (Slavnov-Taylor) identities in the calculations. A modified power counting rule is introduced to consistently estimate and control the contributions of higher loops and higher-dimension operators. The one-loop renormalization group equations of the effective couplings are provided and analyzed. We find that the difference between the results obtained from the direct method and the renormalization group equation method can be quite large when the Higgs scalar boson is far below its decoupling limit. The exact one-loop calculation of d_1 in the renormalizable $SU(2)$ Higgs model is provided to understand such a difference. A better way of calculating at the one-loop level in the framework of the effective theory method is suggested.

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I. INTRODUCTION

To understand nature, the effective field theory (EFT) method is a universal tool, both practical and powerful [1,2]. For example, the Fermi weak interaction theory works quite well at an energy scale far below m_W even before the standard model is established. The effective Hamiltonian method is widely used in B physics enterprises [3]. Although the predictivity of a general EFT is restrained due to the fact that there are an infinite number of permitted operators in its Lagrangian, at a region with an energy lower than the ultraviolet cutoff, these operators can be well organized in terms of their importance to low energy dynamics (i.e., their dimension and the strength of their couplings). For example, among the three groups of effective operators (EO) [4]—the relevant, marginal, and irrelevant ones—only the first two groups dominate the dynamics of low-energy QED and QCD. And in the Fermi theory and B physics theory, only operators equal to and below dimension 6 are important.

As one of the important applications of the EFT method, the effective chiral theories with spontaneous symmetry breaking play a very special role in describing the microscopic world, for example, the QCD chiral perturbation Lagrangian (ChPT) [5], which describes the interactions among hadrons, and the electroweak chiral Lagrangian [6], which describes the interactions among massive vector bosons. As we know, the renormalization group equations (RGE) of an EFT are one of its basic ingredients to describe the behavior of a given system, which, generally speaking, can efficiently sum up the logarithm corrections from the quantum fluctuation of low energy degrees of freedom (DOFs), eliminate or alleviate the renormalization scale and scheme dependences, and improve the perturbation method in strong coupling cases (in QCD, for instance). Compared with the hadronic ChPT, the chiral effective gauge theories (EGT) with massive vector bosons have some special features that make the systematic renormalization difficult. To understand the features, we briefly describe some of facts about the hadronic ChPT first.

In the usual ChPT approach to the low-energy QCD where only massless Goldstone particles are included, the chiral Lagrangian is organized as an expansion in powers of momenta p^2 (such an effective description is good when $p \ll v$, since the interaction vertices are proportional to the power of p/v, which can act as a small quantity for the effective expansion):

$$L^{eff}=L_2+L_4+L_6+\cdots. \quad (1)$$

Each term L_n, in turn, is given by a certain number of operators $O_i^{(n)}$ with low-energy constants $l_i^{(n)}$ that will be determined by the underlying theory (known or unknown):

$$L_n=\sum_i l_i^{(n)}O_i^{(n)}. \quad (2)$$

The general expectation of the importance of an operator is that the lower its order, the more importance it is. Therefore, in the ChPT, L_2 is the most important operator, and it determines the propagators of massless Goldstone bosons and the primary scattering interaction at tree level, which can be expressed as c_2p^2/v^2 [c_2 is a constant of $O(1)$]. At one-loop level, the scattering amplitude will receive the radiative corrections from the loop with two contributions from this vertex and with internal lines of Goldstone bosons. After dropping the divergences of the loop integral, we obtain the finite one-loop contribution of this interaction, which can be expressed as $\alpha[1/(4\pi)^2](p^4/v^4)$ (α is a constant factor determined by the loop and c_2 is of order 1). Such a contribution has a momentum power the same as those of operators in L_4, which can be expressed as $\alpha_0(p^4/v^4)$.

In the ChPT, coincidentally, α_0, as determined from low-energy phenomenologies, such as hadronic scattering and de-

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cay processes, is of the order $1/(4\pi)^2$ [7]. So contributions from L_4 are of the same order as those of the one-loop level of L_2, both with respect to momentum power and to the magnitude of the effective couplings.

So, if we go further to higher order, say two-loop order, then we should include three contributions: (1) the two-loop contributions of pure $O(p^2)$ vertices, (2) the one-loop contribution with one $O(p^2)$ vertex and one $O(p^4)$ vertex, and (3) the tree-level contribution of $O(p^6)$. The first part can be expressed as $\beta_2[1/(4\pi)^4](p^6/v^6)$, the second part can be expressed as $\beta_1[1/(4\pi)^2](p^6/v^6)$, and the third part can be expressed as $\beta_0(p^6/v^6)$. The first part contains the two-loop suppression factor $1/(4\pi)^4$, while the second part contains only the one-loop suppression factor $1/(4\pi)^2$. But due to the fact that β_1 is determined by both c_2 and α_0 with a loop factor $1/(4\pi)^2$, not only by the momentum power but also by the magnitude order controlled by the loop factors, the second part will share the same importance as the first part. We also expect that coincidently, β_0 will have a magnitude like $1/(4\pi)^4$. Thus we expect that such a simple power counting rule (SPCR) will hold for any specified higher order.

But for the EFT with massive vector bosons (and nonlinear interactions), it does not seem easy to take into account the radiative corrections of low-energy quantum DOFs (which should include both the massive vector boson and its corresponding Goldstone boson, and the momentum should not be a small quantity compared with the vacuum expectation value for the application of the theory). The first difficulty concerns the quartic divergence of the theory, which is more manifest when we represent the EGT in the unitary gauge. The propagator of massive vector bosons can be expressed as

$$i\Delta^{\mu\nu}=i\Delta_T^{\mu\nu}+i\Delta_L^{\mu\nu}, \tag{3}$$

$$\Delta_T^{\mu\nu}=\frac{1}{k^2-m_V^2}\left(-g^{\mu\nu}+\frac{k^\mu k^\nu}{k^2}\right), \tag{4}$$

$$\Delta_L^{\mu\nu}=\frac{1}{m_V^2}\frac{k^\mu k^\nu}{k^2}, \tag{5}$$

where Δ_T and Δ_L represent the transverse and longitudinal parts, respectively. The longitudinal part of the propagator can bring quartic divergences and lead to the well-known bad ultraviolet behavior. Two direct consequences of this difficulty are (1) that the quartic divergences will appear in radiative corrections and (2) that low-dimension operators can induce the infinite number of divergences of higher-dimension operators, even at one-loop level. However, in a renormalizable theory, the Higgs model for instance, these two problems do not exist [8]. The quartic divergences produced by the low-energy DOF just cancel exactly with those produced by the Higgs scalar boson, and no extra divergence structure will appear.

Another difficulty, which is also related to the first difficulty, concerns the counting rule. In the gauge theories with spontaneous symmetry breaking, the marginal interaction vertices are proportional to effective couplings (EC) [expressed in both gauge couplings and anomalous couplings (ACs)], not proportional to the momentum power p^2/v^2 as in the hadronic ChPT. Then by directly evaluating the Feynman diagrams, radiative corrections of the ACs [which are determined at matching scale by the ultraviolet dynamics and is unknown to us, and there is no definite reason to assume that they must be as small as $1/(4\pi)^2$] are $1/(4\pi)^2$, not $1/(4\pi)^4$ as expected from the SPCR in the hadronic ChPT. So the native power counting rule cannot properly be used in this case. We know, however, in order to collect and reliably estimate the contributions of higher orders (say, those of higher loops and higher-dimension operators) in terms of magnitude, a power counting rule is needed. So to find a consistent power counting rule for this case is necessary.

To establish a power-counting rule, we should know that for the EGT with a spontaneous symmetry breaking mechanism we have at least two ways to collect and classify operators.

The first way is to collect operators in terms of their dimensions (not by the momentum power p^2/v^2, as in the above case). We can formulate the EGT in unitary gauge [9], then restore the low-energy DOFs with the inverse Stueckelberg transformation, while the ECs are regarded as free parameters, of which the magnitude at the ultraviolet cutoff is determined by the underlying dynamics and the matching conditions. Then, according to the Wilsonian renormalization scheme, EOs can be classified into three groups: relevant operators, marginal operators, and irrelevant operators. The relevant operators have mass dimensions less than the dimension of space-time, and have ECs with positive mass power. The marginal operators have the same dimensions of that of space-time, and have massless ECs. The irrelevant operators have dimensions larger than the dimensions of space-time, and have couplings with negative mass power. By study the running of the ECs, we can determine the importance of operators, which is controlled by the strength of their corresponding ECs. The couplings of the relevant operators will be dependent on the ultraviolet cutoff $\Lambda=\mathrm{UV}$ in positive powers; those of the marginal operators will be logarithmically dependent on the cutoff $\Lambda=\mathrm{UV}$; while those of the irrelevant operators will be dependent on the cutoff $\Lambda=\mathrm{UV}$ in negative powers. If the cutoff $\Lambda=\mathrm{UV}$ is large enough, the irrelevant operators will become unimportant, and the relevant and marginal operators will dominate the low-energy dynamics. Such a conclusion is based on the most general analysis of the behavior of RGEs without assuming the smallness of the ECs of irrelevant operators, as shown in Refs. [4,10]. So equipped with this conclusion in Refs. [4,10], we can truncate the infinite operator towers permitted in the EGT to a specified order.

As we know, the groups of relevant and marginal operators include both the renormalizable operators and anomalous operators (AOs) up to $O(p^4)$. Meanwhile, in the general cases, the relative importance of an operator might be quite different and is mainly determined by the relative magnitude of its EC. For instance, if the coupling is zero or is much much smaller, in principle, we can drop its contributions and regard it as a higher-order correction. If the coupling is much much larger than others, then this operator should be definitely important for the low-energy dynamics. We should then classify it as a lower-order operator to promote its relative importance to the rest of operators. So from our viewpoint, a practical and realistic power counting rule must based on the actual information of the relative magnitude of the ECs.

The second way is to mimic the ChPT by classifying according to the momentum power. In this way, up to $O(p^4)$, without regarding any information on the magnitude of the ECs, these operators are divided into two groups: the renormalizable ones are classified as p^2 order and the anomalous ones are classified as p^4 order. In this way, to classify the gauge kinetic terms as $O(p^2)$ it is somewhat ambiguous for the momentum power counting rule. But it is unlikely not to include the kinetic terms in this p^2 order; otherwise it is impossible to define the propagator of vector bosons. So the dimensionless gauge couplings have to be set to have momentum power. To classify the rest of marginal operators in the group of terms of $O(p^4)$, such a counting rule, borrowed from the hadronic ChPT, implicitly assumes that the strength of their couplings should be of order $1/(4\pi)^2$, so as to guarantee the validity of the power counting rule.

We would like to point out that such assumptions are too strong for a general EGT. In the framework of EFT, the magnitudes of the couplings of an operator is determined at the matching scale. There is no reason to expect that the ACs must be so small. The magnitude of these ACs is related to both the actual value of the matching scale and the underlying dynamics, both of which are unknown to us. As matter of fact, the ACs can receive the tree-level contributions, like in the Higgs model we show in the numerical analysis, in the left-right hand model, etc. Furthermore, even determined at loop level, if the ultraviolet dynamics are of the strong coupling case, as in the Technicolor models, these ACs can be estimated as $[1/(4\pi)^2](g_s^2/g_w^2)$. If g_s is much larger than g_w, the ACs might still be one or two orders larger than the expectation of the SPCR in hadronic ChPT.

So we regard that, in order to be more realistic and be consistent with the EFT method as a general and universal method, we should abandon the second way of the classification of the operators. Before knowing the actual information about the magnitude of the ACs (equivalently, the underlying theories), we will treat all relevant and marginal AOs as operators of $O(p^2)$ order by implicitly assuming all these ACs are of $O(1)$ (this assumption is a more general one, and the assumption of the second way of classification is only one of the specific cases). So we modify the momentum power counting rule to include the ECs of all AOs d_i as momentum p^{-2}, like the coefficient of the gauge kinetic terms $1/g^2$. And in this way, when extracting the Feynman rules directly from the Lagrangian given in (15), the combination of $g^2 d_i$ in the trilinear and quartic couplings is regarded as of the order $O(p^0)\sim O(1)$. Thus, this modified power counting rule will possess the power of the SPCR, and can be applied to estimate and control the contributions of higher-loop and higher-dimension operators, just like that in the hadronic ChPT.

With this modified power counting rule in mind (we will address the unitarity violation problem related to this modified rule in the discussion), below we will study the renormalization of the nonlinear effective $SU(2)$ Lagrangian L^{eff} up to $O(p^4)$ and derive the one-loop RGE of its ECs. We will also numerically study the solutions of these RGEs, and analyze the decoupling and nondecoupling effects of the Higgs boson to those ECs in the effective Lagrangian (EL) L^{eff}. We find that when the Higgs scalar is far below its decoupling limit, our results are significantly different from the results obtained by matching the full theory and EFT directly at the one-loop level [11] (hereby, we call this method the direct method, in contrast with the RGE method). The basic reason for this large difference is that the direct method ignores the contribution of the possible large contributions of the not too heavy Higgs boson, which can considerably affect the ECs through radiative corrections, while the RGE method has taken into account these important effects. To comprehend the underlying reason for this difference, we will provide the exact one-loop formula of the anomalous coupling d_1 in the renormalizable $SU(2)$ theory. However, we find that the cocktail way [which combines the RGE running and the nondecoupling constant term, which can be easily extracted from the direct integrating-out method (DM)] will produce a better prediction that is closer to the exact one-loop calculation.

This paper is organized as follows. In Sec. II, we briefly introduce the renormalizable $SU(2)$ Higgs model, and concentrate on its form in unitary gauge and the quartic divergence term. In Sec. III, the nonlinear effective $SU(2)$ Lagrangian L^{eff} up to $O(p^4)$ introduced, and the initial conditions of ECs is obtained by integrating out the scalar Higgs boson at the tree level. In Sec. VI, we perform the renormalization of the L^{eff} up to $O(p^4)$ in the background field gauge, and by using the Schwinger proper time and heat kernel method, derive the renormalization group equations, so as to sum the leading logarithm contributions of radiative corrections. Section V is devoted to study the numerical solutions of these RGEs in the Higgs scalar decoupling and nondecoupling limits, and to investigate the difference of these methods in the full renormalizable $SU(2)$ theory. We end this paper with some discussions and conclusions.

II. THE RENORMALIZABLE $SU(2)$ HIGGS MODEL

The partition functional of the renormalizable non-Abelian $SU(2)$ Higgs model [12] (here we have not included the gauge fixing term and the ghost term) can be expressed as

$$\mathcal{Z}=\int \mathcal{D}A_\mu^a \mathcal{D}\phi\mathcal{D}\phi^\dagger \exp(i\mathcal{S}[A,\phi,\phi^\dagger]), \tag{6}$$

where the action $\mathcal{S}$ is determined by the following Lagrangian density:

$$\mathcal{L}=-\frac{1}{4g^2}W^a_{\mu\nu}W^{a\mu\nu}+(D\phi)^\dagger\cdot(D\phi)+\mu^2\phi^\dagger\phi-\frac{\lambda}{4}(\phi^\dagger\phi)^2, \tag{7}$$

and the definition of quantities in this Lagrangian is given as

$$W^a_{\mu\nu}=\partial_\mu W^a_\nu-\partial_\nu W^a_\mu+f^{abc}W^b_\mu W^c_\nu, \tag{8}$$

$$D_\mu \phi = \partial_\mu \phi - i W^a_\mu T^a \phi, \tag{9}$$

$$\phi^\dagger = (\phi_1^*, \phi_2^*), \tag{10}$$

where T^a are the generators of the Lie algebra of the $SU(2)$ gauge group.

The spontaneous symmetry breaking is induced by the positive mass squared μ^2 in the Higgs potential. The vacuum expectation value of the Higgs field is given as $|\langle\phi\rangle| = v/\sqrt{2}$. And by eating the corresponding Goldstone boson, the vector bosons W obtain their mass.

The nonlinear form of the Lagrangian given in Eq. (7) is made by changing the variable ϕ:

$$\phi = \frac{1}{\sqrt{2}}(v+\rho)U, \quad U = \exp\left(2i\frac{\xi^a T^a}{v}\right),$$
$$v = 2\sqrt{\frac{\mu^2}{\lambda}}, \tag{11}$$

where the matrix field U is the Goldstone boson field as prescribed by the Goldstone theorem, and the ρ is a massive scalar field. Then it reaches

$$\mathcal{L}' = -\frac{1}{4g^2}W^a_{\mu\nu}W^{a\mu\nu} + \frac{(v+\rho)^2}{4}\mathrm{tr}[(DU)^\dagger \cdot (DU)]$$
$$+\frac{1}{2}\partial\rho\cdot\partial\rho + \frac{1}{2}\mu^2(v+\rho)^2 - \frac{\lambda}{16}(v+\rho)^4. \tag{12}$$

The change of variables induces a determinant factor in the functional integral $\mathcal{Z}$:

$$\mathcal{Z} = \int \mathcal{D}W^a_\mu \mathcal{D}\rho\mathcal{D}\xi^b \exp(i\mathcal{S}'[W,\rho,\xi])$$
$$\times \det\left\{\left(1+\frac{1}{v}\rho\right)\delta(x-y)\right\}. \tag{13}$$

The determinant can be written in the exponential form, and correspondingly the Lagrangian density is modified to

$$\mathcal{L} \to \mathcal{L}' - i\delta(0) ln\left\{1+\frac{1}{v}\rho\right\}. \tag{14}$$

The determinant containing quartic divergences is indispensable and crucial to cancel exactly the quartic divergences brought into by the longitudinal part of the vector boson, and is important in verifying the renormalizability of the Higgs model in the U gauge [8,9].

III. THE NONLINEAR EFFECTIVE $SU(2)$ LAGRANGIAN L^{eff} UP TO $O(P^4)$

In the nonlinear effective $SU(2)$ Lagrangian L^{eff}, only the Goldstone and the vector bosons are included as the effective dynamic freedom at low-energy region. The Lagrangian L^{eff} that includes all permitted operators composed by these light DOFs and respects the assumed Lorentz and gauge symmetries is still renormalizable [13]. Two facts are important for the actual renormalization procedure: (1) The Wilsonian renormalization method [4] and the surface theorem [10] reveal that in the low-energy region, only a few operators play an important role in determining the behavior of the dynamic system at the low-energy region. Such a fact enables us to truncate the infinite divergence tower up to a specified order and to consider the renormalization of the EL order by order. (2) In the dimensional regularization method, the quartic divergences can be expressed to be proportional to the masses in the theory, as the quadratic divergences.

The general effective $SU(2)$ Lagrangian L^{eff} consistent with Lorenz spacetime symmetry, $SU(2)$ gauge symmetry, and the charge, parity, and the combined CP symmetries, can be formulated as

$$\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots, \tag{15}$$

$$\mathcal{L}_2 = -\frac{v^2}{4}\mathrm{tr}[V_\mu V^\mu], \tag{16}$$

$$\mathcal{L}_4 = -\frac{1}{4g^2}W^a_{\mu\nu}W^{a\mu\nu} - i d_1 \mathrm{tr}[W_{\mu\nu}V^\mu V^\nu]$$
$$+ d_2 \mathrm{tr}[V_\mu V_\nu]\mathrm{tr}[V^\mu V^\nu]$$
$$+ d_3 \mathrm{tr}[V_\mu V^\mu]\mathrm{tr}[V_\nu V^\nu], \tag{17}$$
$$\mathcal{L}_6 = \cdots,$$
$$\cdots = \cdots, \tag{18}$$

where $\mathcal{L}_2$ and $\mathcal{L}_4$ represent the relevant and marginal operators in the Wilsonian renormalization method, respectively. Here for the simplicity, below we will omit all the irrelevant operators in our consideration, i.e., dimensional operators higher than $O(p^4)$. m_W is the mass of vector bosons and $m_W = gv/2$. The operators in $\mathcal{L}_2$ and $\mathcal{L}_4$ also form the set of complete operators up to $O(p^4)$ in the usual momentum counting rule. And the dimension (irrelevant) operators higher than $O(p^4)$ order are represented by the dots and omitted here. The auxiliary variable V_μ is defined as

$$V_\mu = U^\dagger D_\mu U, \quad D_\mu U = \partial_\mu U - i W_\mu U \tag{19}$$

to simplify the representation. Due to the following relations of the $SU(2)$ gauge group

$$\mathrm{tr}[T^a T^b T^c T^d] = \frac{1}{8}(\delta^{ab}\delta^{cd} + \delta^{ad}\delta^{bc} - \delta^{ac}\delta^{bd}), \tag{20}$$

the terms, like $\mathrm{tr}[V_\mu V_\nu V^\mu V^\nu]$ and $\mathrm{tr}[V_\mu V^\mu V_\nu V^\nu]$, can be linearly composed by $\mathrm{tr}[V_\mu V_\nu]\mathrm{tr}[V^\mu V^\nu]$ and $\mathrm{tr}[V_\mu V^\mu]\mathrm{tr}[V_\nu V^\nu]$. Since here we do not consider the term that breaks the charge, or parity, or both symmetries, therefore the operators in Eq. (17) are complete and linearly independent.

The ECs of d_i form the parameter space of EFT, and they effectively reflect the dynamics of the underlying theories and the ways of symmetry breaking. Different underlying theories and ways of symmetry breaking will fall into a special point in this effective parameter space. d_i can also be called the ACs if, according to the renormalizable $SU(2)$

gauge theory, they reflect the deviation of the theory from the requirement of renormalizability. When the scale runs from the high-energy region down to the low-energy region, we will obtain a characteristic curve in this parameter space. This curve, if we can measure from the experiments, can help us to figure out the possible underlying theories, as we will do in the Large Hadronic Collider (LHC) by measuring ECs of vector bosons at different energy regions.

When the scalar Higgs boson is heavy and is integrated out, the Higgs model given in Eq. (7) can be effectively described as a special parameter point of the EL given in Eq. (15). At the tree level, it suffices to integrate out the Higgs scalar boson by using its equation of motion, which is expressed in low-energy dynamic DOFs and can be formulated as

$$\rho=\frac{v}{2m_0^2}(DU)^\dagger\cdot(DU)+\cdots, \tag{21}$$

$$m_0^2=\frac{1}{2}\lambda v^2, \tag{22}$$

where m_0 is the mass of Higgs bosons. The omitted terms contain at least four covariant partials and belong to higher-order operators.

By substituting Eq. (21) into Eq. (14) at the matching scale (which is always taken at the scalar mass $\mu=m_0$), the ECs at the tree level are determined as

$$d_1(m_0)=0, d_2(m_0)=0,$$
$$d_3(m_0)=\frac{v^2}{8m_0^2}=\frac{1}{4\lambda},\cdots. \tag{23}$$

In its decoupling limit $m_0\to\infty$ ($\lambda\to\infty$), all these three ECs vanish. Normally, for some theoretical reasons (say, the validity of perturbation theory), λ should not go to ∞, and is usually taken as $O(1)$, as in the standard model. So, d_3 can be quite large compared with other ACs. For other symmetry mechanisms, say Technicolor theories, due to the property of strong couplings, these ECs might be large at the matching scale. So radiative corrections should be taken into account before considering the effects of higher-order operators. Below we will derive the RGEs of the ECs, in order to sum these corrections.

Generally speaking, if a particle does not participate in the process of symmetry breaking and its coupling to low-energy DOF is not proportional to its mass, we know it will not contribute to the ACs up to the $O(p^4)$ order and its effects on the low-energy dynamics will be simply suppressed by its squared mass, according to the decoupling theorem [14].

IV. THE RENORMALIZATION OF L^{eff} AND ITS RENORMALIZATION GROUP EQUATIONS

In the background field method (BFM) [15,16], the number of the Feynman diagrams for the loop corrections can be greatly decreased when compared with the standard Feynman diagram method. Another remarkable advantage is that, in the BFM, each step of calculation is manifestly gauge covariant with reference to the background gauge field, and the Ward identities (Slavnov-Taylor identities in non-Abelian gauge theories)—which are important to restrain the structure of divergences—have been incorporated in the calculation. The Schwinger proper time and heat kernel method [17] by itself is the Feynman integral. Combining with the covariant short distance Taylor expansion [18,19] in coordinate space, the divergent structures can be directly extracted out in the explicit gauge form, and the loop calculation can be simplified to a considerable degree.

A. The quadratic terms of the one-loop Lagrangian

According to the spirit of the BFM, we split the Goldstone and vector bosons into classic and quantum parts, as given below:

$$W\to\overline{W}+\hat{W},\quad U\to\overline{U}\hat{U}. \tag{24}$$

The Stueckelberg transformation [20] combines classic parts $\overline{W}$ and $\overline{U}$ into the Stueckelberg field $\overline{W}^S$

$$\overline{W}^S=\overline{U}^\dagger\overline{W}\overline{U}+i\overline{U}^\dagger\partial\overline{U}, \tag{25}$$

and eliminates the background Goldstone from the EL. The Stueckelberg field is invariant under the background gauge transformation. Such a property guarantees that the following calculation will be unchanged in the background gauge transformation. After finishing the loop calculation, by performing the inverse Stueckelberg transformation (expanding the $\overline{W}^S$ in $\overline{W}$ and $\overline{U}$), the EL can be restored to the form expressed by its low energy DOFs.

As one of the advantages of the BFM, we have the freedom to choose different gauges for the classic and quantum fields, and such freedom can help to further simplify the calculation. The gauge condition for the classic fields can be derived from their classic equation of motion. For quantum fields, we can choose the covariant gauge fixing term as

$$\mathcal{L}_{GF}=-\frac{1}{2g^2}[(D\cdot\hat{W})^a+c_f f^{abc}\overline{W}^{Sb}\cdot\hat{W}^c+f_{ws}\xi^a]^2, \tag{26}$$

where c_f and f_{ws} are determined by requiring the one-loop Lagrangian to have the standard form given in Eq. (31)–(37). Then the condition reads

$$c_f=\frac{1}{2}d_1g^2,\quad f_{ws}=vg^2. \tag{27}$$

The partition functional $\mathcal{Z}$ in the background field gauge can be expressed as

$$\begin{aligned}\mathcal{Z}&=\exp(i\mathcal{S}^{ren}[\overline{W}^S])\\&=\exp(i\mathcal{S}_{tree}[\overline{W}^S]+i\delta\mathcal{S}_{tree}[\overline{W}^S]+i\mathcal{S}_{1\ loop}[\overline{W}^S]+\cdots)\\&=\exp(i\mathcal{S}_{tree}[\overline{W}^S]+i\delta\mathcal{S}_{tree}[\overline{W}^S])\\&\quad\times\int\mathcal{D}\hat{W}_\mu\mathcal{D}\overline{c}\mathcal{D}c\mathcal{D}\xi\exp(i\mathcal{S}[\hat{W},\xi,\overline{c},c;\overline{W}^S]),\end{aligned} \tag{28}$$

where the tree EL $\mathcal{L}_{tree}$ is in the following form

$$\mathcal{L}_{tree}=\frac{v^2}{2}\overline{W}^S\cdot\overline{W}^S-\frac{1}{4g^2}\overline{W}^{Sa}_{\mu\nu}\overline{W}^{S\mu\nu,a}+d_1\frac{1}{4}f^{abc}\overline{W}^{Sa}_{\mu\nu}\overline{W}^{S\mu,b}\overline{W}^{S\nu,c}+d_2\frac{1}{4}\overline{W}^{Sa}\cdot\overline{W}^{Sb}\overline{W}^{Sa}\cdot\overline{W}^{Sb}+d_3\frac{1}{4}(\overline{W}^S\cdot\overline{W}^S)^2+\cdots. \quad (29)$$

The corresponding counter terms $\delta\mathcal{L}_{tree}$ are defined as

$$\delta\mathcal{L}_{tree}=\delta Z_{v^2}\frac{v^2}{2}\overline{W}^{Sa}\cdot\overline{W}^{Sa}-\delta Z_{g^2}\frac{1}{4g^2}\overline{W}^{Sa}_{\mu\nu}\overline{W}^{S\mu\nu,a}+\delta Z_{d_1}d_1\frac{1}{4}f^{abc}\overline{W}^{Sa}_{\mu\nu}\overline{W}^{Sb\ \mu}\overline{W}^{Sc\ \nu}+\delta Z_{d_2}d_2\frac{1}{4}\overline{W}^{Sa}\cdot\overline{W}^{Sb}\overline{W}^{Sa}\cdot\overline{W}^{Sb}+\delta Z_{d_3}d_3\frac{1}{4}(\overline{W}^S\cdot\overline{W}^S)^2+\cdots, \quad (30)$$

where the renormalization constant of the Stueckelberg field $\overline{W}^S$ can always be set to 1.

In the one-loop level, only the quadratic terms of quantum fields are relevant, and they can be cast into the following standard form:

$$\mathcal{L}_{quad}=\frac{1}{2}\hat{W}^a_\mu\Box^{\mu\nu,ab}_{WW}\hat{W}^b_\nu+\frac{1}{2}\xi^a\Box^{ab}_{\xi\ \xi}\xi^b+\overline{c^a}\Box^{ab}_{cc}c^b+\frac{1}{2}\hat{W}^a_\mu\bar{X}^{\mu,ab}\xi^b+\frac{1}{2}\xi^a\vec{X}^{\nu,ab}\hat{W}^b_\nu, \quad (31)$$

$$\Box^{\mu\nu,ab}_{WW}=(D'^{2,ab}+m^2_W\delta^{ab})g^{\mu\nu}-\sigma^{\mu\nu,ab}_{WW}, \quad (32)$$

$$\Box^{ab}_{\xi\xi}=\Box'^{\ ab}_{\xi\xi}+X^{\alpha,ac}d^{cb}_\alpha+X^{\alpha\beta,ac}d^{cd}_\alpha d^{db}_\beta, \quad (33)$$

$$\Box'^{\ ab}_{\xi\xi}=-(d^{2,ab}+\delta^{ab}m^2_W)+\sigma^{ab}_{2,\xi\xi}+\sigma^{ab}_{4,\xi\xi}, \quad (34)$$

$$\Box^{ab}_{cc}=-(D'^{2,ab}+m^2_W\delta^{ab}), \quad (35)$$

$$\bar{X}^{\mu,ab}=\bar{X}^{\mu,ac}_{\alpha\beta}d^{\alpha,cd}d^{\beta,db}+\bar{X}^{\mu\alpha,ac}d^{cb}_\alpha+\bar{X}^{\mu,ab}_{01}+\bar{X}^{\mu,ab}_{03Z}+\partial_\alpha\bar{X}^{\mu\alpha,ab}_{03Y}, \quad (36)$$

$$\vec{X}^{\nu,ab}=\vec{X}^{\nu,ac}_{\alpha\beta}D'^{\alpha,cd}D'^{\beta,db}+\vec{X}^{\nu\alpha,ac}D'^{cb}_\alpha+\vec{X}^{\nu,ab}_{01}+\vec{X}^{\nu,ab}_{03Z}+\partial_\alpha\vec{X}^{\nu\alpha,ab}_{03Y}, \quad (37)$$

where $d_\mu=\partial_\mu-ia_\xi\overline{W}^S_{\mu,G}$, and $D'_\mu=\partial_\mu-ia_W\overline{W}^S_{\mu,G}$. The direction of the harpoon indicates the position of vector bosons, and both the $\bar{X}^{\mu,ab}$ and $\vec{X}^{\nu,ab}$ are defined to act on the right side. It is remarkable that for the gauge fixing terms given in Eq. (26) Goldstone bosons have the same mass as vector bosons.

For the $SU(2)$ EL, the related quantities are defined as

$$\sigma^{\mu\nu ab}_{WW}=2i\overline{W}^{S\mu\nu,ab}_G+\frac{1}{4}d_1^2g^4(\overline{W}^{S\mu,ac}_G\overline{W}^{S\nu,cb}_G-\overline{W}^{Sac}_G\cdot\overline{W}^{Scb}_Gg^{\mu\nu})+id_1g^2(\overline{W}^{S\mu\nu,ab}_G+\overline{F}^{S\mu\nu,ab}_G)-d_2g^2(\overline{W}^{Sa}\cdot\overline{W}^{Sb}g^{\mu\nu}+\overline{W}^{Sc}_\mu\overline{W}^{Sc}_\nu\delta^{ab}+\overline{W}^{Sb}_\mu\overline{W}^{Sa}_\nu)-d_3g^2(\overline{W}^{Sc}\cdot\overline{W}^{Sc}g^{\mu\nu}\delta^{ab}+2\overline{W}^{Sa}_\mu\overline{W}^{Sb}_\nu), \quad (38)$$

$$\sigma^{ab}_{2,\xi\xi}=-a^2_\xi(\overline{W}^S_G\cdot\overline{W}^S_G)^{ab}, \quad (39)$$

$$X^{\alpha\beta,ab}=-\tilde{S}^{\alpha\beta,ab}, \quad (40)$$

$$X^{\alpha,ab}=\tilde{X}^{\alpha,ab}-\partial_\beta\tilde{X}^{\alpha\beta,ab}+2\tilde{S}^{\alpha\beta,ac}\Gamma^{cb}_{\xi,\beta}, \quad (41)$$

$$\sigma^{ab}_{4,\xi\xi}=\tilde{X}^{ab}_4+\tilde{S}^{\alpha\beta,ac}(\partial_\beta\Gamma^{cb}_{\xi,\alpha}-\Gamma^{cd}_{\xi,\alpha}\Gamma^{db}_{\xi,\beta})-\tilde{X}^{\alpha,ac}\Gamma^{cb}_{\xi,\alpha}+\partial_\beta\tilde{X}^{\alpha\beta,ac}\Gamma^{cb}_{\xi,\alpha}, \quad (42)$$

$$\bar{X}^{\mu,ab}_{\alpha\beta}=-\tilde{S}^{\mu,ab}_{\alpha\beta}, \quad (43)$$

$$\bar{X}^{\mu\alpha,ab}=\tilde{X}^{\mu\alpha,ab}_1-\tilde{X}^{\mu\alpha,ab}_2-\partial^\beta\tilde{X}^{\mu,ab}_{\beta\alpha'}g^{\alpha'\alpha}+2\tilde{S}^{\mu,ac}_{\alpha'\beta}\Gamma^{\beta,cb}_\xi g^{\alpha\alpha'}, \quad (44)$$

$$\bar{X}^{\mu,ab}_{01}=\tilde{X}^{\mu,ab}_{01}, \quad (45)$$

$$\bar{X}^{\mu,ab}_{03Z}=\tilde{X}^{\mu,ab}_{03}+\tilde{S}^{\mu,ac}_{\alpha\beta}(\partial^\alpha\Gamma^{\beta,cb}_\xi-\Gamma^{\alpha,cd}_\xi\Gamma^{\beta,db}_\xi)-(\tilde{X}^{\mu\alpha,ac}_1-\tilde{X}^{\mu\alpha,ac}_2)\Gamma^{cb}_{\xi,\alpha}+\partial^\beta\tilde{X}^{\mu,ac}_{\beta\alpha}\Gamma^{\alpha,cb}_\xi, \quad (46)$$

$$\bar{X}^{\mu\alpha,ab}_{03Y}=-\tilde{X}^{\mu\alpha,ab}_2, \quad (47)$$

$$\vec{X}^{\nu,ab}_{\alpha\beta}=-\tilde{S}^{\nu,ba}_{\alpha\beta}, \quad (48)$$

$$\vec{X}^{\nu\alpha,ab}=\tilde{X}^{\nu\alpha,ba}_2-\tilde{X}^{\nu\alpha,ba}_1-\partial^\beta\tilde{X}^{\nu,ba}_{\alpha'\beta}g^{\alpha\alpha'}+2\tilde{S}^{\nu,ca}_{\alpha'\beta}\Gamma^{\beta,cb}_Wg^{\alpha\alpha'}, \quad (49)$$

$$\vec{X}^{\nu,ab}_{01}=\tilde{X}^{\nu,ba}_{01}, \quad (50)$$

$$\vec{X}^{\nu,ab}_{03Z}=\tilde{X}^{\nu,ba}_{03}+\tilde{S}^{\nu,ca}_{\alpha\beta}(\partial^\alpha\Gamma^{\beta,cb}_W-\Gamma^{\alpha,cd}_W\Gamma^{\beta,db}_W)-(\tilde{X}^{\nu\alpha,ca}_2-\tilde{X}^{\nu\alpha,ca}_1)\Gamma^{cb}_{W,\alpha}+\partial^\beta\tilde{X}^{\nu,ca}_{\alpha\beta}\Gamma^{\alpha,cb}_W, \quad (51)$$

$$\vec{X}^{\nu\alpha,ab}_{03Y}=-\tilde{X}^{\nu\alpha,ba}_1, \quad (52)$$

where $\overline{F}^{Sa}_{\mu\nu}=f^{abc}\overline{W}^{Sb}_\mu\overline{W}^{Sc}_\nu$, $\overline{W}^{Sab}_{\mu,G}=if^{acb}\overline{W}^{Sc}_\mu$, $\Gamma^{ab}_{\xi,\mu}=-ia_\xi\overline{W}^{Sab}_{\mu,G}$, and $\Gamma^{ab}_{W,\mu}=-ia_W\overline{W}^{Sab}_{\mu,G}$, with $a_\xi=1/2$ and $a_W=(1+d_1g^2/2)$ (which can be regarded as the effective charge). The a_ξ is determined by the mass term of Eq. (16). For the a_W, the gauge kinetic term in the Eq. (16) and the first AO contribute, while the rest of the AOs do not. To get the above form, we have normalized the vector quantum gauge field by using $\hat{W}/g\to\hat{W}$. When we take the limit $d_i\to 0$, the $\sigma_{WW}^{\mu\nu,ab}$ reaches to its usual form $2iW_G^{\mu\nu,ab}$, as given in the gauge theory without symmetry breaking mechanism.

As in the $U(1)$ case, an auxiliary dimension counting rule is introduced to extract relevant terms up to $O(p^4)$, which reads

$$[\overline{W}_\mu^S]_a=[\partial_\mu]_a=[D_\mu]_a=1,\ [v]_a=0. \tag{53}$$

From this rule, we know

$$[\bar{X}_{\alpha\beta}^{\mu,ab}]_a=[\vec{X}_{\alpha\beta}^{\nu,ab}]_a=[\bar{X}_{01}^{\mu,ab}]_a=[\vec{X}_{01}^{\nu,ab}]_a=1, \tag{54}$$

$$[\sigma_{WW}^{\mu\nu\ ab}]_a=[\sigma_{2,\xi\xi}^{ab}]_a=[X^{\alpha\beta,ab}]_a=[\bar{X}^{\mu\alpha,ab}]_a=[\vec{X}^{\nu\alpha,ab}]_a=[\bar{X}_{03Y}^{\mu\alpha,ab}]_a=[\vec{X}_{03Y}^{\nu\alpha,ab}]_a=2, \tag{55}$$

$$[X^{\alpha,ab}]_a=[\bar{X}_{03Z}^{\mu,ab}]_a=[\vec{X}_{03Z}^{\nu,ab}]_a=3,\ \ [\sigma_{4,\xi\xi}^{ab}]_a=4. \tag{56}$$

We would like to mention that this auxiliary dimension counting rule is to extract those terms with two, three, and four external fields. In the limit that all ACs equal zero, only the $\bar{X}_{01}^{\mu,ab}$, $\vec{X}_{01}^{\nu,ab}$, $\sigma_{WW}^{\mu\nu\ ab}$, and $\sigma_{2,\xi\xi}^{ab}$ do not vanish.

The quantities with tildes are determined from the following prestandard forms [16]

$$\xi^a\Box_{\xi\ \xi}^{ab}\xi^b=-\xi^a(d^{2,ab}+\delta^{ab}m_W^2)\xi^b+\xi^a(\sigma_{2,\xi\xi}^{ab}+\widetilde{X}_4^{ab})\xi^b+\xi^a\widetilde{X}^{\alpha,ab}\partial_\alpha\xi^b+\partial_\alpha\xi^a\widetilde{X}^{\alpha\beta,ab}\partial_\beta\xi^b, \tag{57}$$

$$\begin{aligned}\hat{W}_\mu^a\bar{X}^{\mu,ab}\xi^b&=\xi^a\vec{X}^{\nu,ab}\hat{W}_\nu^b\\&=\partial^\alpha\hat{W}_\mu^a\widetilde{X}_{\alpha\beta}^{\mu,ab}\partial^\beta\xi+\hat{W}_\mu^a\widetilde{X}_1^{\mu\alpha,ab}\partial_\alpha\xi^b\\&\quad+\partial_\alpha\hat{W}_\mu^a\widetilde{X}_2^{\mu\alpha,ab}\xi^b+\hat{W}_\mu^a\widetilde{X}_{01}^{\mu,ab}\xi^b+\hat{W}_\mu^a\widetilde{X}_{03}^{\mu,ab}\xi^b,\end{aligned} \tag{58}$$

and from the EL given in Eq. (15), we get the quantities with tildes expressed in the Stueckelberg field $\overline{W}^S$ and the corresponding strength:

$$\widetilde{X}^{\alpha\beta,ab}=\widetilde{S}^{\alpha\beta,ab}+\widetilde{A}^{\alpha\beta,ab},$$

$$\begin{aligned}\widetilde{S}^{\alpha\beta,ab}&=4\frac{d_2}{v^2}\left(\overline{W}^{Sa}\cdot\overline{W}^{Sb}g^{\alpha\beta}+\overline{W}^{S\alpha,c}\overline{W}^{S\beta,c}\delta^{ab}+\frac{1}{2}H_W^{\alpha\beta,ab}\right)\\&\quad+4\frac{d_3}{v^2}(\overline{W}^{Sc}\cdot\overline{W}^{Sc}g^{\alpha\beta}\delta^{ab}+H_W^{\alpha\beta,ab}),\end{aligned} \tag{59}$$

$$\begin{aligned}\widetilde{A}^{\alpha\beta,ab}&=-i2\frac{d_1}{v^2}\overline{W}_G^{S\alpha\beta,ab}\\&\quad+2\frac{2d_3-d_2}{v^2}(\overline{W}^{S\alpha,a}\overline{W}^{S\beta,b}-\overline{W}^{S\alpha,b}\overline{W}^{S\beta,a}),\end{aligned} \tag{60}$$

$$\begin{aligned}\widetilde{X}^{\alpha,ab}&=-2\frac{d_1}{v^2}(\overline{W}_G^{S\alpha\beta,ad}\overline{W}_{\beta,G}^{S,db}+\overline{W}_{\beta,G}^{S,ad}\overline{W}_G^{S\alpha\beta,db})\\&\quad+4i\frac{d_2}{v^2}\overline{W}^{S\alpha,c}\overline{W}^{S\beta,c}\overline{W}_{\beta,G}^{Sab}+4i\frac{d_3}{v^2}\overline{W}^{Sc}\cdot\overline{W}^{Sc}\overline{W}_G^{S\alpha,ab},\end{aligned} \tag{61}$$

$$\widetilde{X}_4^{ab}=\frac{d_1}{v^2}\overline{W}_G^{S\alpha\beta,ac}\overline{F}_{\alpha\beta,G}^{Scb}, \tag{62}$$

$$\widetilde{X}_{\alpha\beta}^{\mu,ab}=\widetilde{S}_{\alpha\beta}^{\mu,ab}+\widetilde{A}_{\alpha\beta}^{\mu,ab}, \tag{63}$$

$$\widetilde{S}_{\alpha\beta}^{\mu,ab}=i\frac{d_1g}{2v}(2\overline{W}_G^{S\mu,ab}g_{\alpha\beta}-\overline{W}_{\alpha,G}^{Sab}g_\beta^\mu-\overline{W}_{\beta,G}^{Sab}g_\alpha^\mu), \tag{64}$$

$$\widetilde{A}_{\alpha\beta}^{\mu,ab}=-i\frac{d_1g}{2v}(\overline{W}_{\alpha,G}^{Sab}g_\beta^\mu-\overline{W}_{\beta,G}^{Sab}g_\alpha^\mu), \tag{65}$$

$$\begin{aligned}\widetilde{X}_1^{\mu\alpha,ab}&=\frac{d_1g}{v}(\overline{W}_G^{S,ac}\cdot\overline{W}_G^{S,cb}g^{\mu\alpha}-\overline{W}_G^{S\alpha,ac}\overline{W}_G^{S\mu,cb}-i\overline{W}_G^{\alpha\mu,ab})\\&\quad-2\frac{d_2g}{v}(\overline{W}^{Sa}\cdot\overline{W}^{Sb}g^{\mu\alpha}+\overline{W}^{S\alpha,c}\overline{W}^{S\mu,c}\delta^{ab}\\&\quad+\overline{W}^{S\mu,b}\overline{W}^{S\alpha,a})-2\frac{d_3g}{v}(\overline{W}^{Sc}\cdot\overline{W}^{Sc}g^{\mu\alpha}\delta^{ab}\\&\quad+2\overline{W}^{S\alpha,b}\overline{W}^{S\mu,a}),\end{aligned} \tag{66}$$

$$\widetilde{X}_2^{\mu\alpha,ab}=i\frac{d_1g}{v}\overline{F}_G^{S\mu\alpha,ab}, \tag{67}$$

$$\widetilde{X}_{01}^{\mu,ab}=-i\frac{gv}{2}\left(1+\frac{d_1g^2}{2}\right)\overline{W}_G^{S\mu,ab}, \tag{68}$$

$$\begin{aligned}\widetilde{X}_{03}^{\mu,ab}&=\frac{d_1g}{2v}(i\overline{W}_G^{S\mu,ac}\overline{W}_G^{S\alpha,cd}\overline{W}_{\alpha,G}^{Sdb}+i\overline{W}_G^{S\alpha,ac}\overline{W}_G^{S\mu,cd}\overline{W}_{\alpha,G}^{Sdb}\\&\quad+\overline{W}_{\alpha,G}^{Sac}\overline{W}_G^{S\mu\alpha,cb}),\end{aligned} \tag{69}$$

The $H_W^{\alpha\beta,ab}$ is defined as $H_W^{\alpha\beta,ab}=\overline{W}^{S\alpha,a}\overline{W}^{S\beta,b}+\overline{W}^{S\alpha,b}\overline{W}^{S\beta,a}$, which is symmetric on its Lorentz (group) indices.

As we have mentioned, the gauge condition for the background fields can be chosen as being determined by the equation of motion of the Stueckelberg field $\overline{W}^S$, which can be formulated as

$$\partial_\mu \overline{W}^{S\mu\nu,a} - \frac{d_1 g^2}{2}\partial_\mu \overline{F}^{S\mu\nu,a}$$
$$= m_W^2 \overline{W}^{S\nu,a} - f^{abc}\left(1-\frac{d_1 g^2}{2}\right)\overline{W}^{Sb}_\mu \overline{W}^{S\mu\nu,c}$$
$$+\frac{d_1 g^2}{2} f^{abc}\overline{W}^{Sb}_\mu \overline{F}^{S\mu\nu,c} - d_2 g^2 \overline{W}^{S\nu,b}\overline{W}^{S\mu,a}\overline{W}^{Sb}_\mu$$
$$- d_3 g^2 \overline{W}^{S\nu,a}\overline{W}^{S\mu,b}\overline{W}^{Sb}_\mu . \tag{70}$$

From the equation of motion given in Eq. (70), we can get

$$\partial_\nu W^{S\nu,a} = \frac{1}{m_W^2}\partial_\nu\left[f^{abc}\left(1-\frac{d_1 g^2}{2}\right)\overline{W}^{Sb}_\mu \overline{W}^{S\mu\nu,c} + f^{abc}\frac{d_1 g^2}{2}\overline{W}^{Sb}_\mu \overline{F}^{S\mu\nu,c} + d_2 g^2 \overline{W}^{S\nu,b}\overline{W}^{S\mu,a}\overline{W}^{Sb}_\mu + d_3 g^2 \overline{W}^{S\nu,a}\overline{W}^{S\mu,b}\overline{W}^{Sb}_\mu\right]. \tag{71}$$

Then we know that the $(\partial_\mu W^{S\mu,a})^2$ can only contribute to terms at most up to $O(p^6)$. Therefore we simply set $\partial_\mu W^{S\mu,a}=0$ when considering the renormalization up to $O(p^4)$. We have also used the following relations about the Lie algebra:

$$f^{abc}f^{cde}+f^{adc}f^{ceb}+f^{aec}f^{cbd}=0, \tag{72}$$

to simplify the above related expressions.

B. The calculation of the logarithm and traces

The quadratic terms given in the last subsection can be directly calculated by the functional integral, since the integral is Gaussian. Then after integrating out all quantum fields, the $\mathcal{L}_{1\ loop}$ reads

$$\int_x \mathcal{L}_{1\ loop} = i\frac{1}{2}[\mathrm{Tr}\ln\Box_{WW} + \mathrm{Tr}\ln\Box_{\xi\ \xi} + \mathrm{Tr}\ln(1-\vec{X}^\mu\Box^{-1}_{WW;\mu\nu}\tilde{X}^\mu\Box^{-1}_{\xi\ \xi})] - i\ \mathrm{Tr}\,\Box_{\bar{c}c}, \tag{73}$$

where the contribution of the ghost has a different sign due to its anticommutator relation. The Tr is to sum over the Lorentz indices, $\mu\nu$, group indices, ab, and the coordinate space points, x. The operators in the term $\mathrm{Tr}\ln(1-\vec{X}^\mu\Box^{-1}_{WW;\mu\nu}\tilde{X}^\mu\Box^{-1}_{\xi\ \xi})$ are all defined to act on the right side, and such a form reflects the fact that the sequence of integrating out the quantum vector boson and Goldstone fields will not cause any difference in physical results.

The expansion of the logarithm is simply expressed as

$$\langle x|\ln(1-X)|y\rangle = -\langle x|X|y\rangle - \frac{1}{2}\langle x|XX|y\rangle - \frac{1}{3}\langle x|XXX|y\rangle - \frac{1}{4}\langle x|XXXX|y\rangle + \cdots, \tag{74}$$

and here the X should be understood as an operator (a matrix) that acts on the quantum states of the right side.

To calculate of logarithm and traces, it is convenient to conduct the computation in Euclidean space. Below, we will conduct our calculations in Euclidean space. We will use the Schwinger proper time and heat kernel method [17] in coordinate space. In this method, the standard propagators can be expressed as

$$\langle x|\Box^{-1,ab}_{WW;\mu\nu}|y\rangle = \int_0^\infty \frac{d\tau}{(4\pi\tau)^{d/2}}\exp(-m_W^2\tau) \times \exp\left(-\frac{z^2}{4\tau}\right) H^{\mu\nu,ab}_{WW}(x,y;\tau), \tag{75}$$

$$\langle x|\Box'^{-1,ab}_{\xi\ \xi}|y\rangle = \int_0^\infty \frac{d\tau}{(4\pi\tau)^{d/2}}\exp(-m_W^2\tau) \times \exp\left(-\frac{z^2}{4\tau}\right) H_{\xi\xi,ab}(x,y;\tau), \tag{76}$$

where $z=y-x$. The integral over the proper time τ and the factor $\exp[-z^2/(4\tau)]/(4\pi\tau)^{d/2}$ have the effect of separating the quadratic divergent part of the propagator. $H(x,y;\tau)$ is analytic with reference to z and τ, which means that $H(x,y;\tau)$ can be analytically expanded with reference to both z and τ. Then we have

$$H(x,y;\tau) = H_0(x,y) + H_1(x,y)\tau + H_2(x,y)\tau^2 + \cdots, \tag{77}$$

where $H_0(x,y)$, $H_1(x,y)$, and, $H_2(x,y)$ are the Seeley–De Witt coefficients. The coefficient $H_0(x,y)$ is the pure Wilson phase factor, which indicates the phase change of a quantum state when propagating from point y to point x and reads

$$H_0(x,y) = C\exp\left(-\int_y^x \Gamma(z)\cdot dz\right), \tag{78}$$

where $\Gamma(z)$ is the affine connection (dependent on the group representation of the quantum states) defined on the coordinate point z. The coefficient C is related to the spin of the states; for vector bosons, $C=g^{\mu\nu}$ (here and below $g^{\mu\nu}$ should be understood as the metric of Euclidean space, $\delta^{\mu\nu}$), and for scalar bosons, $C=1$. The second Seeley–De Witt $H_1(x,y)$ is related to the σ terms in the D'Alambert operator $D^2-m^2+\sigma$, and $H_1(x,y)=\sigma$. Other coefficients can be found from many sources, say Ref. [17].

The divergence counting rule of the integral over the coordinate space x and the proper time τ can be established as

$$[z^\mu]_d = 1, \quad [\tau]_d = -2. \tag{79}$$

Using Eq. (74), the two propagators defined in Eq. (75) and Eq. (76), and the divergence and momentum counting rule given in Eq. (53) and (79), up to $O(p^4)$ [omitting those higher order divergent structures, terms in $O(p^6)$, $O(p^8)$, and so on], we can get the following divergent terms:

$$\bar{\epsilon}\,\mathrm{Tr}\ln\Box_{WW}=-\int_x\left\{m_W^2\mathrm{tr}[g_{\mu\nu}\sigma_{WW}^{\mu\nu}]+\frac{8}{3}\left(\frac{1}{4}\Gamma^a_{W,\mu\nu}\Gamma_W^{\mu\nu,a}\right)+\frac{1}{2}\mathrm{tr}[\sigma_{WW}^{\mu\mu'}g_{\mu'\nu'}\sigma_{WW}^{\nu'\nu}g_{\mu\nu}]\right\}, \quad (80)$$

$$\bar{\epsilon}\,\mathrm{Tr}\ln\Box_{\bar{c}c}=-\int_x\left[\frac{2}{3}\left(\frac{1}{4}\Gamma^a_{W,\mu\nu}\Gamma_W^{\mu\nu,a}\right)\right], \quad (81)$$

$$\bar{\epsilon}\,\mathrm{Tr}\ln\Box_{\xi\xi}=-\int_x\left\{-m_W^2\mathrm{tr}[\sigma_{2,\xi\xi}]-m_W^2\mathrm{tr}[\sigma_{4,\xi\xi}]+\frac{2}{3}\left(\frac{1}{4}\Gamma^a_{\xi,\mu\nu}\Gamma_\xi^{\mu\nu,a}\right)+\frac{1}{2}\mathrm{tr}[\sigma_{2,\xi\xi}\sigma_{2,\xi\xi}]-\frac{1}{4}m_W^4X^{\alpha\beta,ab}g_{\alpha\beta}\delta^{ab}+\frac{1}{2}m_W^2\mathrm{tr}[X^{\alpha\beta,ab}g_{\alpha\beta}\sigma_{2,\xi\xi}^{ba}]-\frac{1}{16}m_W^4g^{\alpha\beta\alpha'\beta'}tr[X^{\alpha\beta,ab}X^{\alpha'\beta',ba}]\right\}, \quad (82)$$

$$\mathrm{Tr}\ln(1-\bar{X}^\mu\Box^{-1}_{WW;\mu\nu}\bar{X}^\mu\Box^{-1}_{\xi\xi})=-\int_x\frac{1}{\bar{\epsilon}}(p4t+p3t+p2t), \quad (83)$$

where $1/\bar{\epsilon}=(2/\epsilon-\gamma_E+\ln4\pi^2)/(16\pi^2)$, γ_E is the Euler constant, and $\epsilon=4-d$. The $\Gamma_{\mu\nu}$ is the field strength tensor corresponding to the affine connection Γ_μ.

We would like to comment on the difference of right-hand side (r.h.s.) of the $\mathrm{Tr}\ln\Box_{WW}$ and $\mathrm{Tr}\ln\Box_{\xi\xi}$. The minus in the $\mathrm{Tr}\ln\Box_{\xi\xi}$ is due to the different definition in the standard form about $\sigma_{WW}^{\mu\nu}$ and $\sigma_{\xi\xi}$. We have used the dimensional regularization scheme and the modified minimal subtraction scheme to extract the divergent structures in this step.

The first three $\mathrm{Tr}\ln\Box$ is obtained straightforward from the heat kernel method, and the last $\mathrm{Tr}\ln(1-\bar{X}^\mu\Box^{-1}_{WW;\mu\nu}\bar{X}^\mu\Box^{-1}_{\xi\xi})$ is provided first by us. The $p4t$ represents the contributions of four propagators $\mathrm{tr}(\bar{X}\Box^{-1}_{WW}\bar{X}\Box'^{-1}_{\xi\xi}\bar{X}\Box^{-1}_{WW}\bar{X}\Box'^{-1}_{\xi\xi})$, which reads

$$p4t=\frac{g_{\mu\nu}g_{\mu'\nu'}}{6}\left[\frac{g^{\alpha\beta\alpha'\beta'}}{4}\mathrm{tr}[2\bar{X}^\mu_{\alpha\beta}\bar{X}^\nu_{\alpha'\beta'}\bar{X}^{\mu'}_{01}\bar{X}^{\nu'}_{01}+2\bar{X}^\mu_{01}\bar{X}^\nu_{\alpha\beta}\bar{X}^{\mu'}_{\alpha'\beta'}\bar{X}^{\nu'}_{01}+\bar{X}^\mu_{\alpha\beta}\bar{X}^\nu_{01}\bar{X}^{\mu'}_{\alpha'\beta'}\bar{X}^{\nu'}_{01}+\bar{X}^\mu_{01}\bar{X}^\nu_{\alpha\beta}\bar{X}^{\mu'}_{01}\bar{X}^{\nu'}_{\alpha'\beta'}]+m_W^2\frac{g^{\alpha\beta\alpha'\beta'\alpha''\beta''}}{4}\mathrm{tr}[\bar{X}^\mu_{\alpha\beta}\bar{X}^\nu_{\alpha'\beta'}\bar{X}^{\mu'}_{\alpha''\beta''}\bar{X}^{\nu'}_{01}+\bar{X}^\mu_{\alpha\beta}\bar{X}^\nu_{\alpha'\beta'}\bar{X}^{\mu'}_{01}\bar{X}^{\nu'}_{\alpha''\beta''}]+m_W^4\frac{g^{\alpha\beta\alpha'\beta'\delta\gamma\delta'\gamma'}}{32}\mathrm{tr}[\bar{X}^\mu_{\alpha\beta}\bar{X}^\nu_{\alpha'\beta'}\bar{X}^{\mu'}_{\delta\gamma}\bar{X}^{\nu'}_{\delta'\gamma'}]\right]. \quad (84)$$

$p3t$ represents the contributions of three propagators $\mathrm{tr}(\bar{X}\Box^{-1}_{WW}\bar{X}\Box'^{-1}_{\xi\xi}X_{\alpha\beta}d^\alpha d^\beta\Box'^{-1}_{\xi\xi})$, which reads

$$p3t=-m_W^4\frac{g^{\alpha\beta\alpha'\beta'\delta\gamma}}{16}g_{\mu\nu}\mathrm{tr}[\bar{X}^\mu_{\alpha\beta}\bar{X}^\nu_{\alpha'\beta'}X_{\delta\gamma}]-\frac{m_W^2}{8}g^{\alpha\beta\alpha'\beta'}g_{\mu\nu}\mathrm{tr}[\bar{X}^\mu_{01}\bar{X}^\nu_{\alpha\beta}X_{\alpha'\beta'}+\bar{X}^\mu_{\alpha\beta}\bar{X}^\nu_{01}X_{\alpha'\beta'}]-\frac{1}{4}g^{\alpha\beta}g_{\mu\nu}\mathrm{tr}[\bar{X}^\mu_{01}\bar{X}^\nu_{01}X_{\alpha\beta}]. \quad (85)$$

$p2t$ represents the contributions of two propagators $\mathrm{tr}(\bar{X}\Box^{-1}_{WW}\bar{X}\Box'^{-1}_{\xi\xi})$, which can be further divided into six groups:

$$p2t=t_{AA}+t_{AB}+t_{AC}+t_{BB}+t_{BC}+t_{CC}, \quad (86)$$

$$t_{AA}=\frac{1}{8}g^{\alpha\beta\alpha'\beta'}m_W^4g^{\mu\nu}\mathrm{tr}[\bar{X}^\mu_{\alpha\beta}\bar{X}^\nu_{\alpha'\beta'}]-\frac{m_W^2g^{\mu\nu}}{4}\left(\frac{g^{\alpha\beta\alpha'\beta'\delta\gamma}}{6}-\frac{g^{\alpha\beta}g^{\alpha'\beta'\delta\gamma}}{2}-\frac{g^{\alpha'\beta'}g^{\alpha\beta\delta\gamma}}{2}+g^{\alpha\beta}g^{\alpha'\beta'}g^{\delta\gamma}\right)\mathrm{tr}[\bar{X}^\mu_{\alpha\beta}D_\delta D_\gamma\bar{X}^\nu_{\alpha'\beta'}]-\frac{m_W^2g^{\alpha\beta\alpha'\beta'}}{8}[g_{\mu\nu}\mathrm{tr}[\bar{X}^\mu_{\alpha\beta}\bar{X}^\nu_{\alpha'\beta'}H_{1,\xi\xi}]+g_{\mu\mu'}g_{\nu\nu'}\mathrm{tr}[\bar{X}^\mu_{\alpha\beta}H_{1,WW}\mu'\nu'\bar{X}^\nu_{\alpha'\beta'}]],$$

$$t_{AB}=-\frac{m_W^2g_{\alpha'\alpha''}g_{\mu\nu}}{2}\left(g^{\alpha\beta}g^{\alpha'\beta'}-\frac{1}{2}g^{\alpha\beta\alpha'\beta'}\right)\mathrm{tr}[\bar{X}^\mu_{\alpha\beta}D_{\beta'}\bar{X}^{\nu\alpha''}-\bar{X}^{\mu\alpha''}D_{\beta'}\bar{X}^\nu_{\alpha\beta}], \quad (87)$$

$$2t_{AC}=\frac{m_W^2g^{\alpha\beta}g_{\mu\nu}}{2}\mathrm{tr}[\bar{X}^{\mu}_{\alpha\beta}\bar{X}^{\nu}_{01}+\bar{X}^{\mu}_{01}\bar{X}^{\nu}_{\alpha\beta}+\bar{X}^{\mu}_{\alpha\beta}\bar{X}^{\nu}_{03Z}+\bar{X}^{\mu}_{03Z}\bar{X}^{\nu}_{\alpha\beta}-\partial_{\alpha'}\bar{X}^{\mu}_{\alpha\beta}\bar{X}^{\nu\alpha'}_{03Y}-\bar{X}^{\mu\alpha'}_{03Y}\partial_{\alpha'}\bar{X}^{\nu}_{\alpha\beta}]$$
$$-\frac{1}{4}g^{\alpha\beta}\mathrm{tr}[g_{\mu\nu}\bar{X}^{\mu}_{01}\bar{X}^{\nu}_{\alpha\beta}H_{1,\xi\xi}+g_{\mu\mu'}g_{\nu\nu'}\bar{X}^{\mu}_{\alpha\beta}H^{\mu\nu'\nu'}_{1,WW}\bar{X}^{\nu}_{01}]$$
$$+g_{\mu\nu}\left(\frac{1}{6}g^{\alpha\beta\alpha'\beta'}-\frac{1}{4}g^{\alpha\beta}g^{\alpha'\beta'}\right)\mathrm{tr}[\bar{X}^{\mu}_{\alpha\beta}D_{\alpha'}D_{\beta'}\bar{X}^{\nu}_{01}+\bar{X}^{\mu}_{01}D_{\alpha'}D_{\beta'}\bar{X}^{\nu}_{\alpha\beta}], \tag{88}$$

$$t_{BB}=\frac{m_W^2g_{\mu\nu}g_{\alpha\beta}}{2}\mathrm{tr}[\bar{X}^{\mu\alpha}\bar{X}^{\nu\beta}], \tag{89}$$

$$t_{BC}=\frac{g^{\alpha\beta}g_{\alpha\alpha'}g_{\mu\nu}}{2}\mathrm{tr}[\bar{X}^{\mu\alpha'}D_{\beta}\bar{X}^{\nu}_{01}-\bar{X}^{\mu}_{01}D_{\beta}\bar{X}^{\nu\alpha'}], \tag{90}$$

$$t_{CC}=g_{\mu\nu}\mathrm{tr}[\bar{X}^{\mu}_{01}\bar{X}^{\nu}_{01}+\bar{X}^{\mu}_{01}\bar{X}^{\nu}_{03Z}+\bar{X}^{\mu}_{03Z}\bar{X}^{\nu}_{01}-\partial_{\alpha'}\bar{X}^{\mu}_{01}\bar{X}^{\nu\alpha'}_{03Y}-\bar{X}^{\mu\alpha'}_{03Y}\partial_{\alpha'}\bar{X}^{\nu}_{01}], \tag{91}$$

where the trace is made to sum over the group indices, and the covariant differentials is defined as

$$\bar{X}D\bar{X}=\bar{X}\partial\bar{X}+\bar{X}\Gamma_W\bar{X}-\bar{X}\bar{X}\Gamma_{\xi}, \tag{92}$$

$$\bar{X}DD\bar{X}=\bar{X}\partial\partial\bar{X}+\bar{X}\Gamma_W\Gamma_W\bar{X}+\bar{X}\bar{X}\Gamma_{\xi}\Gamma_{\xi}-2\bar{X}\Gamma_W\bar{X}\Gamma_{\xi}+2\bar{X}\Gamma_W\partial\bar{X}-2\bar{X}\partial\bar{X}\Gamma_{\xi}+\bar{X}\partial\Gamma_W\bar{X}-\bar{X}\bar{X}\partial\Gamma_{\xi}. \tag{93}$$

In the $p2t$ terms, we have dropped those terms with gauge field strength terms, due to the reason that a term with the symmetric Lorentz indices contracting with the antisymmetric Lorentz indices of gauge strength vanishes. The tensors $g^{\alpha\beta\gamma\delta}$ and $g^{\alpha\beta\gamma\delta\mu\nu}$ are symmetric on all indices and defined as

$$g^{\alpha\beta\gamma\delta}=g^{\alpha\beta}g^{\gamma\delta}+g^{\alpha\gamma}g^{\beta\delta}+g^{\alpha\delta}g^{\beta\gamma}, \tag{94}$$

$$g^{\alpha\beta\gamma\delta\mu\nu}=g^{\alpha\beta}g^{\gamma\delta\mu\nu}+g^{\alpha\gamma}g^{\beta\delta\mu\nu}+g^{\alpha\delta}g^{\gamma\beta\mu\nu}+g^{\alpha\mu}g^{\beta\gamma\delta\nu}+g^{\alpha\nu}g^{\beta\gamma\delta\mu}. \tag{95}$$

To get $p4t$, $p3t$, and $p2t$, we have used the covariant short-distance expansion technology [18,19] and the integral over the proper time and coordinate space. The Dirichlet integral formula has been used to perform the integral over proper times of particles in loops

$$I=\int_0^1 f(t_1+t_2+\cdots+t_n)t_1^{\alpha_1-1}t_2^{\alpha_2-1}\cdots t_n^{\alpha_n-1}dt_1dt_2\cdots dt_n$$
$$=\frac{\Gamma(\alpha_1)\Gamma(\alpha_2)\cdots\Gamma(\alpha_n)}{\Gamma\left(\sum_{i=1}^n \alpha_i\right)}\int_0^1 f(t)t^{(\sum_{i=1}^n \alpha_i-1)}dt. \tag{96}$$

We would like to comment on the covariant short-distance expansion technology: to formulate the quadratic form into the standard form we have prescribed in Eqs. (31)–(37) can greatly reduce the labor to extract the one loop divergences, while the form given in Ref. [19] is not easy to use. The equivalence of these two forms can be easily proved by using the partial integral. As we have pointed out, the standard form given by us has the advantage of reflecting the fact that the order of integrating out the quantum vector boson and Goldstone fields has no any dynamic significance.

C. The renormalization group equations

Substituting Eqs. (38)–(69) into Eqs. (80)–(91), with somewhat tedious algebraic manipulation, we can construct the counterterms and extract the renormalization constants.[1] The renormalization constants yield the following RGEs:

$$\frac{dg^2}{dt}=\frac{g^4}{8\pi^2}\left[-\frac{29}{4}-\frac{20d_1g^2}{3}-\frac{23d_1^2g^4}{24}\right], \tag{97}$$

$$\frac{dv}{dt}=\frac{v}{16\pi^2}\left[\frac{3g^2}{2}+\left(\frac{5d_1}{2}-5d_2-\frac{35d_3}{2}\right)g^4+\frac{13d_1^2g^6}{16}\right], \tag{98}$$

$$\frac{dd_1}{dt}=\frac{1}{8\pi^2}\left\{-\frac{1}{12}+\left(\frac{-43d_1}{6}-\frac{5d_2}{2}+5d_3\right)g^2-\frac{91d_1^2g^4}{12}-\frac{29d_1^3g^6}{24}\right\}, \tag{99}$$

$$\frac{dd_2}{dt}=\frac{1}{8\pi^2}\left\{-\frac{1}{12}+\left(\frac{5d_1}{16}+8d_2+3d_3\right)g^2+\left[\frac{15d_1^2}{8}+6d_2^2+5d_2d_3+d_3^2+d_1\left(\frac{45d_2}{2}+4d_3\right)\right]g^4+\left[\frac{-11d_1^3}{8}+d_1^2\left(\frac{53d_2}{4}+\frac{21d_3}{4}\right)\right]g^6-\frac{43d_1^4g^8}{24}\right\}, \tag{100}$$

[1]The MATHEMATICA package for the whole calculation in this paper is available by request to yanqs@ mail.ihep.ac.cn and yanqs@post.kek.jp

$$\frac{dd_3}{dt}=\frac{1}{8\pi^2}\left\{-\frac{1}{24}+\left(\frac{-3d_1}{16}-\frac{9d_2}{2}-4d_3\right)g^2+\left[\frac{-67d_1^2}{16}+\frac{9d_2^2}{4}+d_1\left(-14d_2-\frac{15d_3}{2}\right)+13d_2d_3+\frac{25d_3^2}{2}\right]g^4+\left[\frac{-39d_1^3}{8}+d_1^2\left(\frac{-79d_2}{8}-\frac{45d_3}{4}\right)\right]g^6-\frac{19d_1^4g^8}{12}\right\}. \tag{101}$$

Concerning the RGEs given in Eqs. (97)–(101), it is remarkable that the direct method will only get part of the result of the RGE method, which is contributed by the Goldstone boson and indicated by the constant terms independent of d_i, $i=1,2,3$ in the r.h.s. of RGEs of d_i, $i=1,2,3$. The remaining terms of the RGEs take into account not only the effect of the Goldstone boson ξ, but also that of vector bosons $\hat{W}$ and that of their mixing terms. Similar to the $U(1)$ case, there are no terms like d_2^2, d_3^2, and d_2d_3, which are due to the cancellation between the vector and Goldstone bosons.

Another remarkable feature is that the d_i always appear with d_ig^2. Such a feature is understandable if we extract the Feynman rules directly from the effective Lagrangian. According to our modified counting rule, terms d_ig^2 are of $O(1)$, so the power of these terms should also be of $O(1)$ and be kept in the beta functions.

In order to compare and contrast, we formulate the results of the direct method in the RGE form, which read

$$\frac{dg^2}{dt}=\frac{g^4}{8\pi^2}\left[-\frac{29}{4}\right], \tag{102}$$

$$\frac{dv}{dt}=\frac{v}{16\pi^2}\left[\frac{3\ g^2}{2}\right], \tag{103}$$

$$\frac{dd_1}{dt}=\frac{1}{8\pi^2}\left[-\frac{1}{12}\right], \tag{104}$$

$$\frac{dd_2}{dt}=\frac{1}{8\pi^2}\left[-\frac{1}{12}\right], \tag{105}$$

$$\frac{dd_3}{dt}=\frac{1}{8\pi^2}\left[-\frac{1}{24}\right]. \tag{106}$$

Under the assumption that d_i are of size $1/(4\pi)^2$, then terms of d_i in the beta functions given in Eq. (101) should belong to two-loop effects, so we can neglect them and get Eq. (106). The underlying reason to represent the contributions of Higgs bosons in the RGE form might be related with the fact that the full theory is renormalizable and the divergences generated by the Higgs loop should be canceled out exactly by those generated by the Goldstone bosons.

To extract the divergent structures, we have used the following relations of the $SU(2)$ gauge group (the equation of motion of $\overline{W}^S$, $\partial\cdot\overline{W}^S=0$, has also been used):

$$\mathrm{tr}[W_G^{S\mu}W_G^{S\nu}W_{\mu,G}^{S}W_{\nu,G}^{S}]=2\overline{W}^{Sa}\cdot\overline{W}^{Sb}\overline{W}^{Sa}\cdot\overline{W}^{Sb}, \tag{107}$$

$$\mathrm{tr}[W_G^{S\mu}W_{\mu,G}^{S}W_G^{S\nu}W_{\nu,G}^{S}]=\overline{W}^{Sa}\cdot\overline{W}^{Sb}\overline{W}^{Sa}\cdot\overline{W}^{Sb}+(\overline{W}^{S}\cdot\overline{W}^{S})^2, \tag{108}$$

$$\begin{aligned}H_{\mu\nu}^{a}H^{\mu\nu,a}=&W_{\mu\nu}^{a}W^{a\mu\nu}-2f^{abc}\overline{W}_{\mu\nu}^{Sa}\overline{W}^{S\mu,b}\overline{W}^{S\nu,c}\\&+(\overline{W}^{S}\cdot\overline{W}^{S})^2\\&-\overline{W}^{Sa}\cdot\overline{W}^{Sb}\overline{W}^{Sa}\cdot\overline{W}^{Sb},\end{aligned}$$

$$F_{\mu\nu}^{Sa}F^{S\mu\nu,a}=-\overline{W}^{Sa}\cdot\overline{W}^{Sb}\overline{W}^{Sa}\cdot\overline{W}^{Sb}+(\overline{W}^{S}\cdot\overline{W}^{S})^2 \tag{109}$$

where the variable $H_{\mu\nu}^a$ is invariant when exchanging its Lorentz indices μ and ν, and is defined as $H_{\mu\nu}^a=\partial_\mu W_\nu^{Sa}+\partial_\nu W_\mu^{Sa}$.

The term $5d_3g^2$ in the right-hand side of the RGE of d_1 is quite remarkable: the coefficient 5 mainly comes from $\mathrm{Tr}\ln\Box_{WW}$, which contributes 8; t_{CC} contributes -2, and $\mathrm{Tr}\ln\Box_{\xi\xi}$ contributes -1. When the Higgs boson is not too heavy [say, λ is near $O(1)$], the coupling d_3 ($d_3=1/\lambda$) can reach order 0.1 or 0.01. This term can then switch the sign of $d_1(m_W)$ from positive to negative. This fact will explain why $d_1(m_W)$ changes its sign when the Higgs boson is not too heavy, as we will show below.

V. NUMERICAL ANALYSIS

We concentrate on the Higgs scalar boson effects to the ECs d_i, $i=1,2,3$. To simplify the analysis, we mimic the standard model by choosing the mass of vector boson m_W to be 80 GeV. The Higgs scalar boson is assumed to be heavier than the vector bosons W. The initial condition for the coupling g and the vacuum expectation value v is fixed at the lower boundary point, $\mu=m_W$. The coupling $g(m_W)$ is chosen to satisfy

$$\alpha_g=\frac{g^2}{4\pi}=\frac{1}{30}, \tag{110}$$

which gives $g(m_W)=0.65$ and the vacuum expectation value is then fixed by $m_W=\frac{1}{2}gv$, which gives $v(m_W)=247$. While the initial condition for d_i, $i=1,2,3$ is chosen to be fixed at the matching scale, $\mu=m_0$, as given in Eq. (23).

Below we will compare the results obtained from the direct method and the RGE method. In order to compare the difference with the DM (where the tree-level Higgs contributions are neglected), we set the initial conditions of d_i vanishing at the matching scale. As we know, the scalar boson effect includes both the decoupling mass squared suppressed part as shown in Eq. (23) and the nondecoupling logarithm part. So we consider the following three cases to trace the change of roles of these two competing parts: (1) the light scalar boson case, with $m_0=150$ GeV, where the decoupling mass squared suppressed part dominates; (2) the

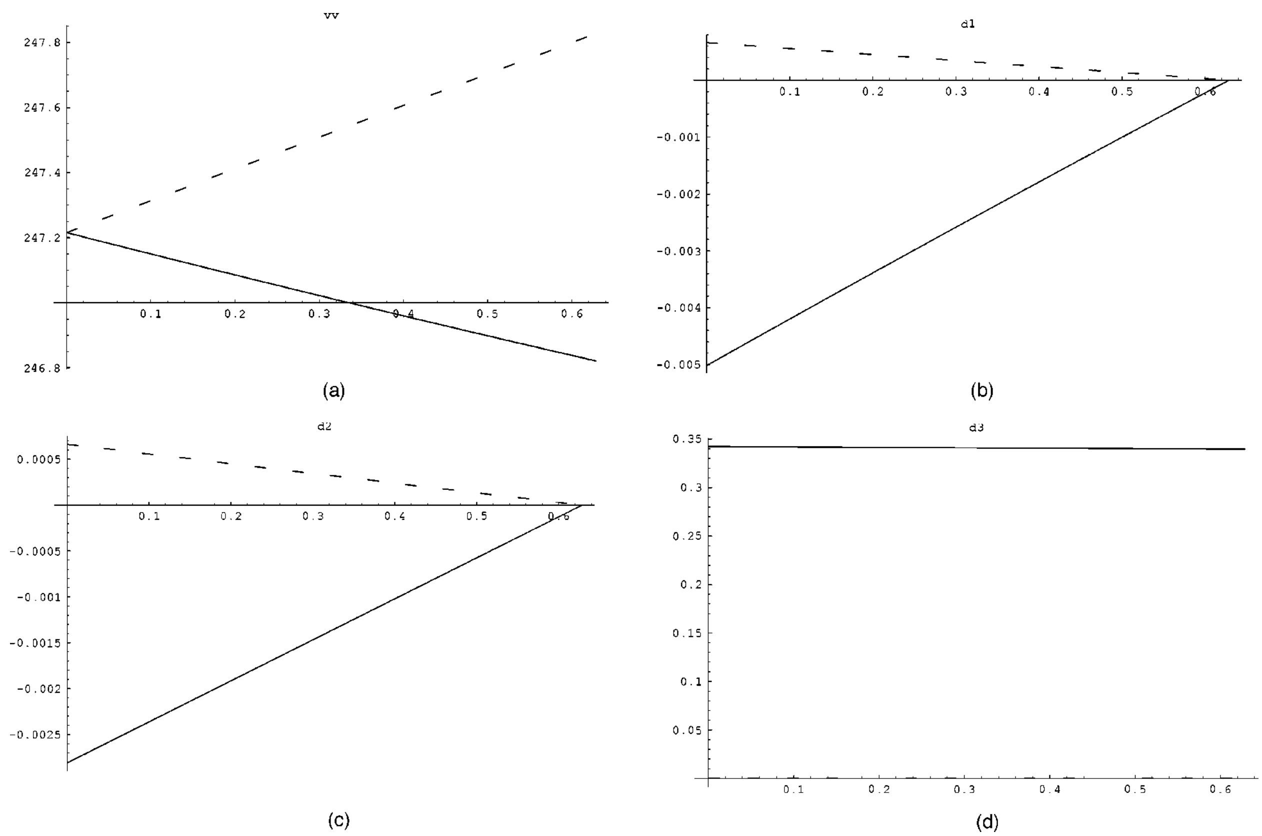

FIG. 1. The varying of v, d_1, d_2, and d_3 with the running scale $t[t=\ln(m_0/m_W)]$. The matching scale is the mass of Higgs scalar boson, which is taken to be $m_0=150$ GeV. The solid lines are the results of the RGE method, while the dashed ones are those of the direct method.

not too heavy scalar boson case, with $m_0=450$ GeV, where both contributions are important; (3) the very massive scalar boson case, with $m_0=900$ TeV, where the nondecoupling logarithm part dominates.

Figure 1 is devoted to the first case, Fig. 2 to the second case, Fig. 3 to the third case. In all three cases, the magnitude of the $d_2(m_W)$ is about 10^{-3} in both methods, and the difference between these two methods is as follows: (1) in the first case, the result of RGE method is about 400% larger than that of the direct method; (2) in the second case 50% smaller; (3) in the third case, there is a negligible difference.

$d_1(m_W)$ can reach 10^{-2} in the RGE method, one order larger than in the direct method, as in the first case when the Higgs scalar boson is quite small. Even when the Higgs boson is medium heavy, as in the second case, the results of these two methods are also quite different. Near the decoupling limit, the prediction of the RGE method improves that of the direct method up to 5%–10%.

Due to its initial values at the matching scale, the $d_3(m_W)$ could have different magnitudes in these three cases, 10^{-1}, 10^{-2}, and 10^{-3}, respectively. In total, the differences of these two methods are dramatic. The figures of d_3 indicate that the tree-level contribution is much larger than the one-loop contribution, and the difference is measured by orders.

The difference of the running of g can be neglected in these two methods so we have not depicted it. The underlying reason is explicit, in that the contributions of the constants of its β function is much larger than those of the ACs.

From these figures, we see the tendency that the difference of δd_1 between these two methods is larger when the Higgs scalar boson is further below its decoupling limit, and vice versa. The underlying reason for this behavior is related to the initial value d_3 at the matching scale and the related terms dependent on d_3 in the RGEs given in Eq. (97)–(101).

It is constructive to compare the predictions of these two different methods of the same theoretical framework (the direct method and the renormalization group equation method of the same framework of the effective theory) with the third method, i.e., the perturbation calculation of the renormalizable $SU(2)$ Higgs model. According to our numerical analysis, the anomalous coupling d_1 is greatly different in these two methods, so below we will concentrate on the comparison of the results for d_1, while leaving a complete comparison of all d_i to our next work.

The one-loop effects of the Higgs boson on d_1 can be obtained by the background field method in coordinate space (in coordinate space, the momentum dependence is neglected, though can be restored by using the RGE method), and the result is given as

$$d_1(m_W)=-\frac{1}{16\pi^2}\,\frac{17r^3-189r^2+423r-251-6(r^2-33r+48)r\log r}{72(r-1)^3}, \tag{111}$$

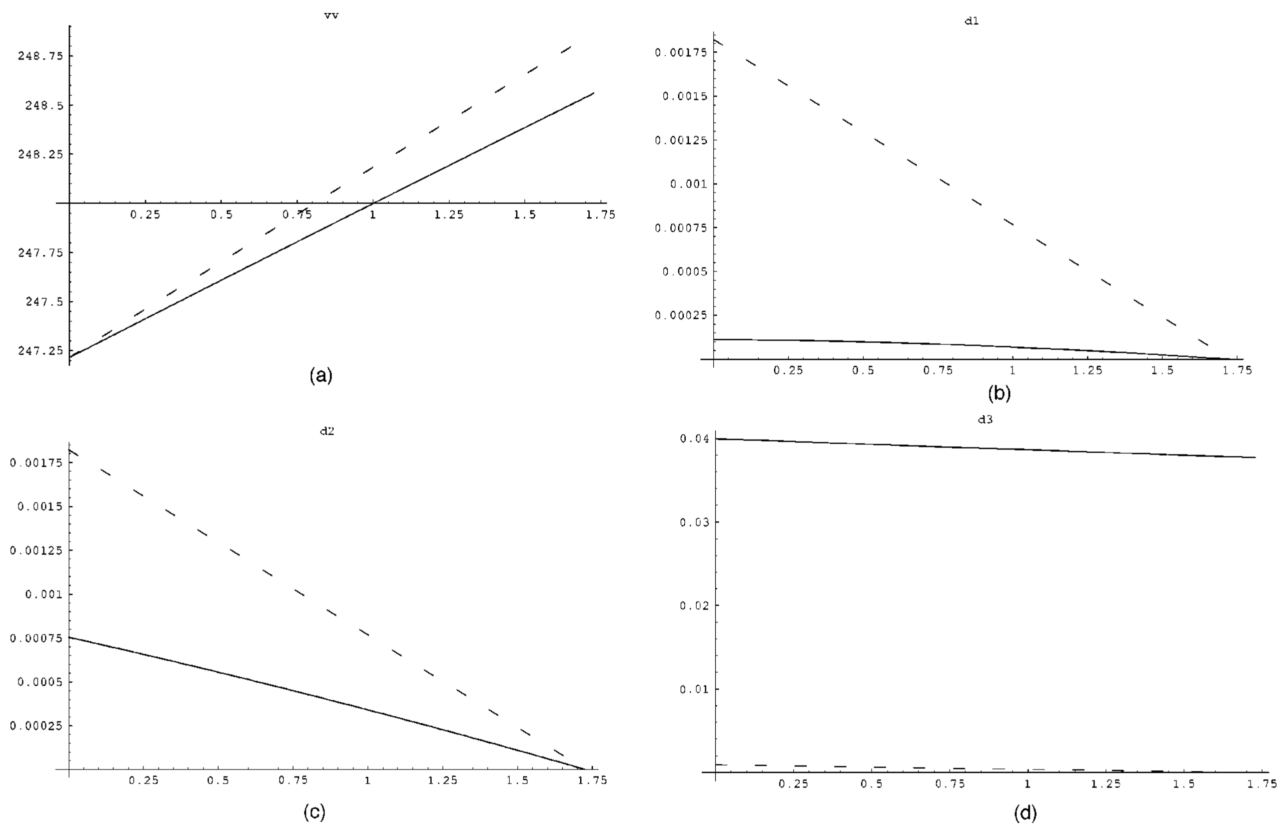

FIG. 2. The varying of v, d_1, d_2, and d_3 with the running scale $t[t=\ln(m_0/m_W)]$. The matching scale is the mass of Higgs scalar boson, which is taken to be $m_0=450$ GeV. The solid lines are the results of the RGE method, while the dashed ones are those of the direct method.

where the variable r is defined as m_H^2/m_W^2. In the decoupling limit, the nondecoupling part is

$$d_1^{non}(m_W)=\frac{1}{16\pi^2}\left[\frac{1}{12}\log r-\frac{17}{72}\right], \tag{112}$$

The difference between the complete one-loop result and the nondecoupling part, i.e., the decoupling part (which vanishes if the mass ratio r approaches infinity), is given as

$$\delta d_1(m_W) =-\frac{1}{16\pi^2}\frac{-23r^2+62r-39+(30r^2-45r-1)\log r}{12(r-1)^3}. \tag{113}$$

Figure 4 is devoted for comparison of the $d_1(m_W)$ calculated by these three methods. First, we consider the tree-level matching conditions in the Fig. 4(a), where the constant term $-17/72$ in Eq. (112) and also the corresponding term in Eq. (111) are omitted. Then when compared with the complete one-loop result, it is obvious that the decoupling limit (where when only the logarithmic part is taken into account) is not always the most important part, especially when the Higgs boson is relatively light. From the complete one-loop result given in Eq. (111), we find that the RGE method takes into account not only the term of $\log r$, but also $\log r/r$, while the direct method only the term $\log r$. In the case when the Higgs boson is heavy and only the log terms play a major part in trilinear couplings, then both these two methods yield almost the same prediction. In the tree-level matching condition, roughly speaking, the RGE method gives a better prediction close to the exact one-loop calculation than the DM does.

In the one-loop level, the nondecoupling constant term $-17/72$ is important. To see the effect of this constant term, in Fig. 4(b) for both the DM and the complete one-loop calculation we take this constant term into account while for the RGE method we still omit it. Then from Fig. 4(b), we know that this constant term can considerably affect the magnitude of d_1.

If we take into account the nondecoupling constant term $-17/72$ in all these three methods, the corresponding curves in Fig. 4(a) will only shift downward in parallel. However, the shapes will not change, and the prediction of the RGE method will still be better than that of the DM, similar to the tree-level case.

From the two cases we have considered, the tree-level and one-loop level matching conditions, we can conclude that the cocktail way, i.e., the nondecoupling constant term (which can be easily extracted from the DM) plus the running RGE, will produce a better prediction closer to the exact one-loop calculation. We can also read from Fig. 4(a) that the effects of higher-order operators are not of much importance in the RGE method. The RGE method can yield a good prediction for a relative light Higgs boson in a wide range. Only when

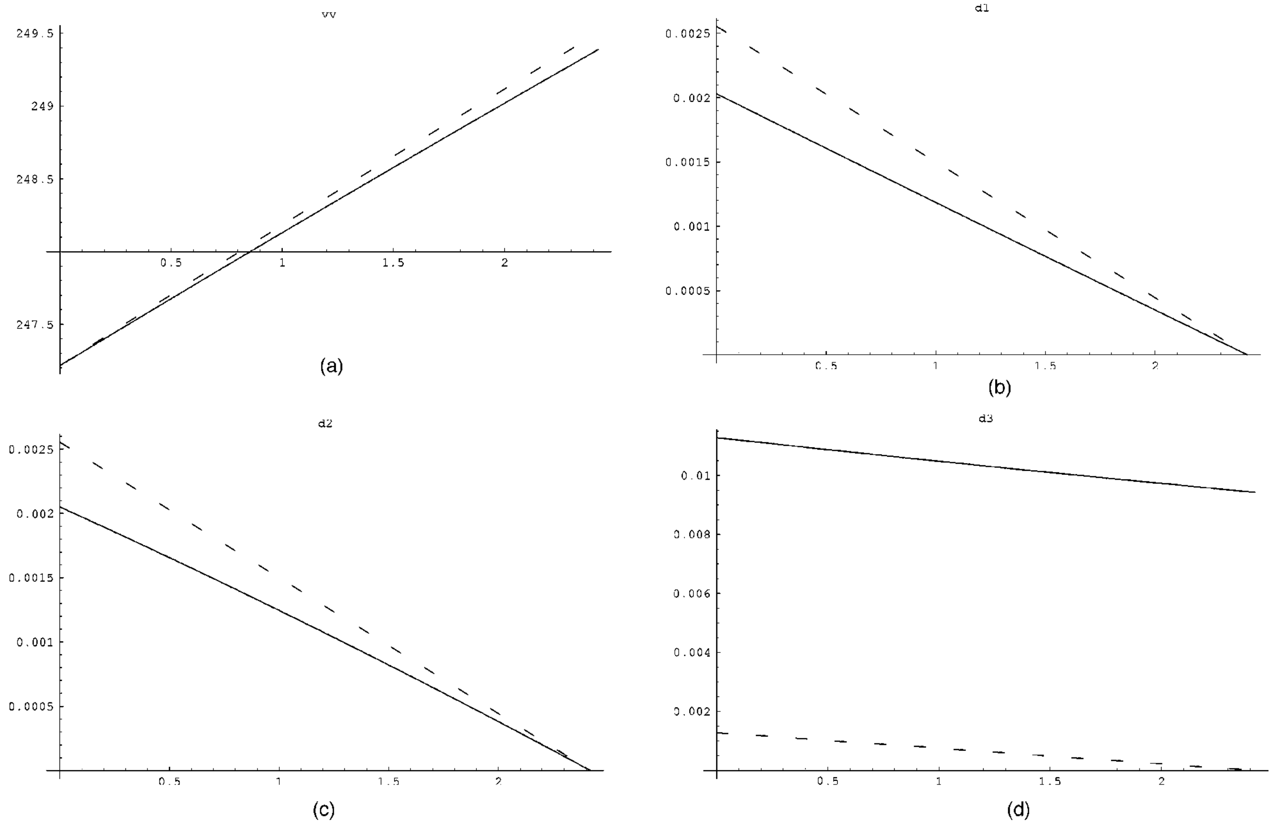

FIG. 3. The varying of v, d_1, d_2, and d_3 with the running scale $t[t=\ln(m_0/m_W)]$. The matching scale is the mass of Higgs scalar boson, which is taken to be $m_0=900$ GeV. The solid lines are the results of the RGE method, while the dashed ones are those of the direct method.

the Higgs boson is near $2m_W$ or so might the deviation of the prediction of RGE from the exact one-loop calculation become considerable.

To understand the reason why d_1 has changed its sign in the lower-energy region, it is also constructive to provide the relevant Feynman diagrams. The terms proportional to d_3 in the beta function of d_1 are equivalent to the contribution of the diagram (in unitary gauge) given in Fig. 5. While in the underlying theory, this diagram corresponds to the two diagrams (in unitary gauge) given in Fig. 6. By contracting the Higgs line to a point, the Feynman diagrams in Fig. 6 reduce to the diagram in Fig. 5. It is remarkable that the contribution from the vector-Higgs mixing plays the most important and constructive part in the light Higgs boson region, as has been shown by the dotted line in Fig. 4(a).

VI. DISCUSSIONS AND CONCLUSIONS

In this paper, we have studied the renormalization of the nonlinear effective $SU(2)$ Lagrangian with spontaneous symmetry breaking and derived its RGEs. Compared with the $U(1)$ case [21], the non-Abelian case is much more complicated. In the $SU(2)$ case, the gauge coupling and the ACs up to $O(p^4)$ all develop by the quantum fluctuation low-energy DOFs. We also have comparatively studied the results of the direct method and the RGE method in the framework of the effective field theory. From the numerical analysis, we see that the results of the two methods are very different when the Higgs scalar boson is far below its decoupling limit. The underlying reason is related to the initial value of d_3 at the matching scale and to the radiative correction of all low-energy DOF (both the Goldstone and vector bosons), which contributes to the d_3 terms in Eqs. (97)–(101). We also provide the one-loop result in the renormalizable $SU(2)$ theory to comprehend the difference. In the one-loop level, it seems the combination of the DM, and the RGE method can yield a better theoretical prediction.

Normally, when the Higgs boson is very light compared with the mass of vector bosons, the higher-dimension operators, for instance, those belonging to the $O(p^6)$ order, might play some considerable parts and it might be not good to use EFT to describe the full theory, since the Wilsonian renormalization [4] and the surface theorem given by Ref. [10] require that the low-energy scale μ_{IR} is lower enough than the UV cutoff μ_{UV}. But here we see, for the medium heavy Higgs boson (say, from $m_0=200$ GeV to $m_0=600$ GeV), it is still appropriate to use it, though the cocktail method from both the DM and the RGE method is recommended.

According to the SPCR, you might regard this result as not reliable, since according to the naive chiral counting rule, the radiative correction of ACs should be at the $O(p^6)$ order, and should be comparable with two-loop corrections of $O(p^2)$ operators. It seems unlikely to change the signs of d_1. But we would like to point out that this naive power counting rule can only hold when the condition $m_{\text{Higgs}}\sim 4\pi v\sim 12v$ is satisfied, i.e., the tree-level contribution of the Higgs boson is comparable to or even much smaller than the one-loop

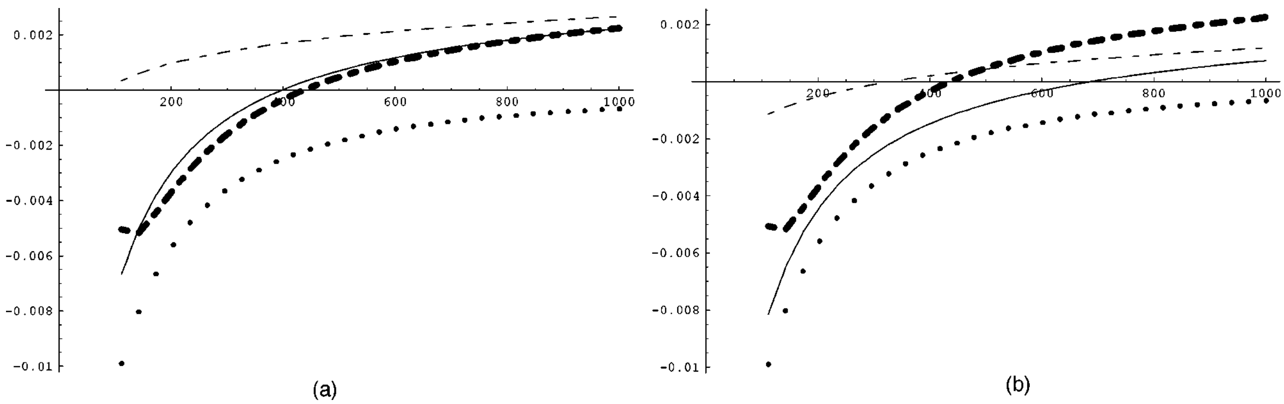

FIG. 4. The comparison of the DM and RGE method in the effective theory and the complete one-loop computation in the renormalizable $SU(2)$ Higgs theory. The X axis is the Higgs boson mass, m_H, and the unit is GeV; the Y axis is d_1. The thin dashed line is the result of DM, the thick dashed line the result of RGE, the solid line the complete one-loop computation, and the dotted line the contribution of vector-Higgs boson mixing term to d_1. The difference between (a) and (b) is that the results of the DM and the exact one-loop have taken into account the nonlogarithmic nondecoupling part $-17/72$, while that of the RGE method has not.

level corrections. For the medium heavy Higgs and light Higgs cases, this power counting rule cannot be used as a reliable guide to understand the accurate calculations from the RGE method. With the modified power counting rule, such a fact is quite easy to understand.

In the RGE method, it becomes quite transparent that the effects of heavy DOF on the low-energy dynamics are related to two factors: (1) the mass of the heavy particle, which determines the matching scale μ_{UV}, and (2) the initial values of ACs at the matching scale determined by integrating out the heavy particle, which are related to the spin of the heavy particle and the strength of its couplings to the low-energy DOFs. If a heavy field does not participate in the process of symmetry breaking, by using the decoupling theorem [14], its effects can be estimated.

To establish the modified power counting rule and to derive the RGEs, we have assumed that all ACs are of $O(1)$. By assuming ACs of $O(1)$, the EL might be limited in the realistic application, due to the fact that the amplitude of the longitudinal components of vector boson (Goldstone boson) scattering processes at higher-energy regions might violate the unitarity condition once the momentum of the vector boson goes a little higher than the mass of vector bosons. Considering the fact that the parameter space of the effective theories should be composed by both the ultraviolet cutoff Λ_{UV} and the ACs at that scale, the violation of unitarity just imposes a helpful correlation on the matching scale and the ACs. If the magnitude of ACs is smaller, then the Λ_{UV} can be larger, vice versa. So our assumption and the RGE method have relatively more flexibility to match an unknown underlying theory from m_W to $4\pi v$ than the specific case by assuming ACs are tiny and the cutoff is at $4\pi v$ (which might miss some important effects of underlying theory when the cutoff is not that large, as we have shown in the Higgs model). As are matter of fact, for the case when the ACs are large, before the unitarity condition is actually violated, new particles or new resonances might have been found. Therefore, new effective theories should be formulated to include new particles, and new RGEs should be derived. So there is no necessity for us to worry about the problem of the unitarity violation.

As we know, there are several ways for the $SU(2)$ to break into its subgroups; $SU(2)$ breaks to $U(1)$ [22], for instance. In this paper, we only assume that the symmetry is broken from a local one to a global one, where all components of vector boson have obtained the same mass. For the way of $SU(2)$ breaking to $U(1)$, the EL will be more com-

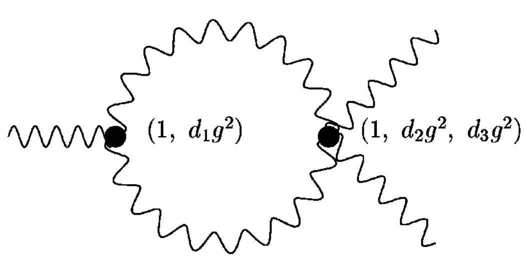

FIG. 5. The related Feynman diagram (in unitary gauge) in the effective theory that contributes to the trilinear couplings.

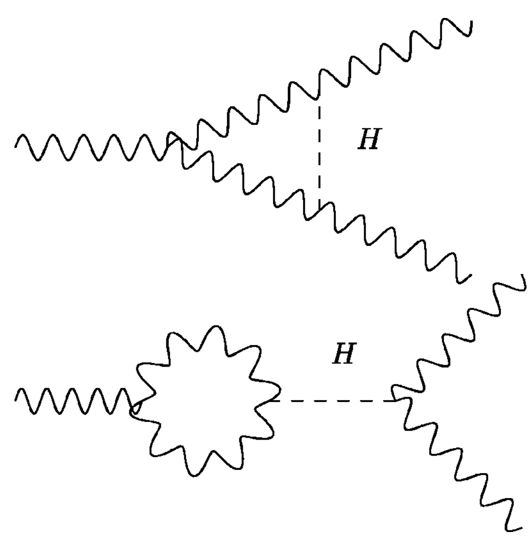

FIG. 6. The related Feynman diagram (in unitary gauge) in the renormalizable theory that contributes to the trilinear couplings.

plicated. Several patterns of symmetry breaking will be discussed in our next paper [23] when we consider the renormalization of electroweak chiral Lagrangian.

Meanwhile, for the sake of simplicity, no fermion field is taken into account, which might introduce terms of anomaly. Also, we have not included all of terms breaking the charge, parity, and both symmetries. If included, the above procedure will be more complicated due to the properties of the complete antisymmetric tensor $\epsilon^{\mu\nu\delta\gamma}$. However, in principle, we can still make the renormalization procedure order by order even for the complexity.

The renormalization procedure in this paper can easily be extended to study the renormalization of the nonlinear sigma model with $SU(N_f)$ symmetry [5], which has a very important role in describing low-energy hadronic physics. We will apply the related conceptions and methods to the renormalization of the electroweak chiral effective Lagrangian and the QCD chiral Lagrangian in our future work [23].

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APPENDIX: THE INTEGRAL IN COORDINATE SPACE

Here we provide some necessary integrals in coordinate space. The basic formula we will use to conduct the integrals include the following ones:

$$\int d^{d-1}\Omega = 2\frac{\pi^{d/2}}{\Gamma(d/2)}, \tag{A1}$$

$$\int_0^\infty ds s^{z-1}\exp[-z] = 2\int_0^\infty dt t^{2z-1}\exp[-t^2] = \Gamma(z). \tag{A2}$$

The basic trick for the integral over coordinate space is given as below:

$$z^\mu z^\nu \to \frac{g^{\mu\nu}}{d} z^2, \tag{A3}$$

$$z^\mu z^\nu z^\alpha z^\beta \to \frac{g^{\mu\nu\alpha\beta}}{d(d+2)} z^4, \tag{A4}$$

$$z^\mu z^\nu z^\alpha z^\beta z^\delta z^\gamma \to \frac{g^{\mu\nu\alpha\beta\delta\gamma}}{d(d+2)(d+4)} z^6, \tag{A5}$$

$$\cdots = \cdots. \tag{A6}$$

As shorthand, we define

$$T^{\alpha\beta}(z,\lambda) = \frac{z^\alpha z^\beta}{4\lambda^2} - \frac{g^{\alpha\beta}}{2\lambda}. \tag{A7}$$

Case A, integrals with one propagator:

$$I^1 = \lim_{y\to x}\int \frac{d\lambda}{(4\pi\lambda)^{d/2}}\exp[-m^2\lambda]\exp\left[-\frac{(y-x)^2}{4\lambda}\right]K^1, \tag{A8}$$

where $z=y-x$, and K^1 can be regarded as the kernel of this integral transformation. When K^1 is known and the integral can be performed, we have the quartic divergent integral,

$$K^1 = T^{\alpha\beta}(z,\lambda), \tag{A9}$$

$$I^1 = -\frac{g^{\alpha\beta}}{2}\frac{1}{(4\pi)^{d/2}}(m^2)^{d/2}\Gamma(-d/2); \tag{A10}$$

the quadratic divergent integral,

$$K^1 = T^{\alpha\beta}(z,\lambda)\lambda, \tag{A11}$$

$$I^1 = -\frac{g^{\alpha\beta}}{2}\frac{1}{(4\pi)^{d/2}}(m^2)^{d/2-1}\Gamma(1-d/2); \tag{A12}$$

and the logarithmic divergent integral,

$$K^1 = T^{\alpha\beta}(z,\lambda)\lambda^2, \tag{A13}$$

$$I^1 = -\frac{g^{\alpha\beta}}{2}\frac{1}{(4\pi)^{d/2}}(m^2)^{d/2-2}\Gamma(2-d/2). \tag{A14}$$

Case B, the integrals with two propagators:

$$I^2 = \int d^d z \frac{d\lambda_1}{(4\pi\lambda_1)^{d/2}}\frac{d\lambda_2}{(4\pi\lambda_2)^{d/2}}\exp[-m_1^2\lambda_1 - m_2^2\lambda_2] \times\exp\left[-z^2\left(\frac{1}{4\lambda_1}+\frac{1}{4\lambda_2}\right)\right]K^2. \tag{A15}$$

We have quartic divergent integrals,

$$K^2 = T^{\alpha\beta}(z,\lambda_1)T^{\alpha'\beta'}(z,\lambda_2),$$

$$I^2 = \frac{g^{\alpha\beta\alpha'\beta'}}{4}\frac{1}{(4\pi)^{d/2}}\Gamma\left(-\frac{d}{2}\right)\int_0^1 \delta(1-x_1-x_2) \times(m_1^2 x_1 + m_2^2 x_2)^{d/2}, \tag{A16}$$

the quadratic divergent integrals,

$$K^2 = \left\{\frac{z^\alpha z^\beta}{4\lambda_1\lambda_2}, T^{\alpha\beta}(z,\lambda_1), T^{\alpha\beta}(z,\lambda_1)\frac{z_2^{\alpha'} z_2^{\beta'}}{2\lambda_2}, T^{\alpha\beta}(z,\lambda_1)T^{\alpha'\beta'}(z,\lambda_2)z^\delta z^\gamma, T^{\alpha\beta}(z,\lambda_1)T^{\alpha'\beta'}(z,\lambda_2)\lambda_1\right\}, \tag{A17}$$

$$I^2=\frac{1}{(4\pi)^{d/2}}\Gamma(1-d/2)\int_0^1\delta(1-x_1-x_2)(m_1^2x_1+m_2^2x_2)^{d/2-1}\times\left\{\frac{g^{\alpha\beta}}{2},-\frac{g^{\alpha\beta}}{2},\frac{1}{2}(g^{\alpha\beta\alpha'\beta'}x_1-g^{\alpha\beta}g^{\alpha'\beta'}),\frac{1}{2}(g^{\alpha\beta\alpha'\beta'\delta\gamma}x_1x_2-g^{\alpha\beta}g^{\alpha'\beta'\delta\gamma}x_1-g^{\alpha'\beta'}g^{\alpha\beta\delta\gamma}x_2+g^{\alpha\beta}g^{\alpha'\beta'}g^{\delta\gamma}),\frac{1}{4}g^{\alpha\beta\alpha'\beta'}x_1\right\}, \quad (A18)$$

the logarithmic divergent integrals,

$$K^2=\left\{1,\frac{z^\alpha z^\beta}{2\lambda_1},T^{\alpha\beta}(z,\lambda_1)z^{\alpha'}z^{\beta'},T^{\alpha\beta}(z,\lambda_1)\lambda_1,T^{\alpha\beta}(z,\lambda_1)\lambda_2\right\}, \quad (A19)$$

$$I^2=\frac{1}{(4\pi)^{d/2}}\Gamma(2-d/2)\int_0^1\delta(1-x_1-x_2)(m_1^2x_1+m_2^2x_2)^{d/2-2}\times\left\{1,g^{\alpha\beta}x_2,g^{\alpha\beta}x_1,(x_2^2g^{\alpha\beta\alpha'\beta'}-x_2g^{\alpha\beta}g^{\alpha'\beta'}),-\frac{1}{2}g^{\alpha\beta}x_1,-\frac{1}{2}g^{\alpha\beta}x_2\right\}. \quad (A20)$$

By interchanging symmetry between the indices $i=1$ and $i=2$, the rest of integrals can be easily obtained and are omitted here.

Case C, the integrals with three propagators,

$$I^3=\int d^dz_1d^dz_2\prod_{i=1}^{3}\frac{d\lambda_i}{(4\pi\lambda_i)^{d/2}}\exp\left[-\sum_{i=1}^{3}m_i^2\lambda_i\right]\times\exp\left[-\sum_{i=1}^{3}\frac{z_i^2}{4\lambda_i}\right]K^3, \quad (A21)$$

where $z_3=-(z_1+z_2)$. We have the quartic divergent integral,

$$K^3=T^{\alpha\beta}(z_1,\lambda_1)T^{\alpha'\beta'}(z_2,\lambda_2)T^{\delta\gamma}(z_3,\lambda_3), \quad (A22)$$

$$I^3=-\frac{g^{\alpha\beta\alpha'\beta'\delta\gamma}}{8}\frac{1}{(4\pi)^{d/2}}\Gamma(-d/2)\times\int_0^1dx_1dx_2dx_3\left(\sum_{i=1}^{3}x_im_i^2\right)^{d/2}\delta\left(1-\sum_{i=1}^{3}x_i\right), \quad (A23)$$

the quadratic divergent integral,

$$K^3=T^{\alpha\beta}(z_1,\lambda_1)T^{\alpha'\beta'}(z_2,\lambda_2), \quad (A24)$$

$$I^3=\frac{g^{\alpha\beta\alpha'\beta'}}{4}\frac{1}{(4\pi)^{d/2}}\Gamma(1-d/2)\times\int_0^1dx_1dx_2dx_3\left(\sum_{i=1}^{3}x_im_i^2\right)^{d/2-1}\delta\left(1-\sum_{i=1}^{3}x_i\right); \quad (A25)$$

the logarithmic divergent integral,

$$K^3=T^{\alpha'\beta'}(z_2,\lambda_2), \quad (A26)$$

$$I^3=-\frac{g^{\alpha'\beta'}}{2}\frac{1}{(4\pi)^{d/2}}\Gamma(2-d/2)\times\int_0^1dx_1dx_2dx_3\left(\sum_{i=1}^{3}x_im_i^2\right)^{d/2-2}\delta\left(1-\sum_{i=1}^{3}x_i\right). \quad (A27)$$

By interchanging symmetry among the indices $i=1$, $i=2$, and $i=3$, the rest of integrals can be easily obtained and are omitted here.

Case D, the integrals with four propagators:

$$I^4=\int d^dz_1d^dz_2d^dz_3\prod_{i=1}^{4}\frac{d\lambda_i}{(4\pi\lambda_i)^{d/2}}\exp\left(-\sum_{i=1}^{4}m_i^2\lambda_i\right)\times\exp\left(-\sum_{i=1}^{4}\frac{z_i^2}{4\lambda_i}K^4\right), \quad (A28)$$

where $z_4=-(z_1+z_2+z_3)$. We have the quartic divergent integral,

$$K^4=T^{\alpha\beta}(z_1,\lambda_1)T^{\alpha'\beta'}(z_2,\lambda_2)T^{\delta\gamma}(z_3,\lambda_3)T^{\delta'\gamma'}(z_4,\lambda_4), \quad (A29)$$

$$I^4=\frac{g^{\alpha\beta\alpha'\beta'\delta\gamma\delta'\gamma'}}{16}\frac{1}{(4\pi)^{d/2}}\Gamma(-d/2)\times\int_0^1\prod_{i=1}^{4}dx_i\left(\sum_{i=1}^{4}x_im_i^2\right)^{d/2}\delta\left(1-\sum_{i=1}^{4}x_i\right), \quad (A30)$$

the quadratic divergent integral,

$$K^4=T^{\alpha\beta}(z_1,\lambda_1)T^{\alpha'\beta'}(z_2,\lambda_2)T^{\delta\gamma}(z_3,\lambda_3), \tag{A31}$$

$$I^4=-\frac{g^{\alpha\beta\alpha'\beta'\delta\gamma}}{8}\frac{1}{(4\pi)^{d/2-1}}\Gamma(1-d/2)\times\int_0^1\prod_{i=1}^4 dx_i\left(\sum_{i=1}^4 x_i m_i^2\right)^{d/2-1}\delta\left(1-\sum_{i=1}^4 x_i\right), \tag{A32}$$

the logarithmic divergent integral,

$$K^4=T^{\alpha'\beta'}(z_2,\lambda_2)T^{\delta\gamma}(z_3,\lambda_3), \tag{A33}$$

$$I^4=\frac{g^{\alpha'\beta'\delta\gamma}}{4}\frac{1}{(4\pi)^{d/2-1}}\Gamma(2-d/2)\times\int_0^1\prod_{i=1}^4 dx_i\left(\sum_{i=1}^4 x_i m_i^2\right)^{d/2-2}\delta\left(1-\sum_{i=1}^4 x_i\right). \tag{A34}$$

By using the interchange symmetry among the indices $i=1$, $i=2$, $i=3$, and $i=4$, the rest of integrals can be easily obtained and are omitted here.

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